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THE THERMAL UTILIZATION FACTOR IN A TIGHT LATTICE

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The thermal utilization factor is calculated in the diffusion approximation for a lattice of thick cylindrical fuel blocks. It is shown that when the lattice possesses sufficient symmetry (when similar blocks are located at the vertices of a hexagon, a square, or an equilateral triangle) the method of equivalent cells is a sufficiently good approximation even if the blocks are very close together. The solutions are given in a form such that nondiffusion corrections may be introduced.

INTRODUCTION

The thermal utilization coefficient in a heterogeneous nuclear reactor is given by the following expression:

$$\Theta = \frac{\text{the number of thermal neutrons absorbed in the blocks of a cell}}{\text{the number of thermal neutrons absorbed in the whole cell}}$$

The quantity Θ depends primarily on the geometry and the nuclear properties of the fuel blocks and thermal neutron moderator; to a somewhat lesser degree, it depends on the way the epithermal neutron density varies from cell to cell (i.e., on the "Laplacian" κ^2) and within a cell (on the relation between the cell dimensions and the moderation length). For practical purposes, as has been shown by Galanin [1], in calculating Θ one may neglect both the macroscopic (from cell to cell) and the microscopic (within a cell) inhomogeneities in the epithermal neutron field. Both in the work of Galanin and in the earlier work of Pomeranchuk and Akhiezer, however, the blocks are assumed "filament-like," i.e., the thermal neutron field in the blocks is considered symmetric. Actually, however, there occur cases in which the blocks occupy half the volume of the cell. This is particularly true for uranium-water lattices, although a tight lattice may occur also in a gas-cooled graphite uranium reactor if the block is pierced by gas channels and therefore has a large diameter. The question therefore arises as to whether the "filament-like" block method or some similar "equivalent cell" method (Wigner-Seitz cell) is applicable to lattices with thick blocks, when the neutron field in the region of the blocks may be asymmetric.

In the present work Θ is calculated for a tight lattice on the assumption that the neutron field is described by a diffusion equation. The solution, however, is given in a form which allows the introduction of corrections for deviations from the diffusion approximation in the block and in its vicinity. It is shown that for lattices possessing a high order of symmetry (for instance when similar blocks are located at the vertices of equilateral triangles, i.e., for a triangular lattice) the corrections to the formula obtained by the "filament-like" block method or that obtained by the Wigner-Seitz cell method are extremely small even if the block volume is greater than that of the moderator.

1. Thermal Neutron Field in the Moderator and Θ

Let us assume that the neutron moderation density is constant and numerically equal to the capture cross section per unit volume of the moderator Σ^c. Then the thermal neutron flux $\Phi(\bar{r})$ in the moderator satisfies the diffusion equation

$$(1 - L^2\Delta)\Phi = 1, \tag{1.1}$$

where $L^2 = \frac{\lambda^{tr}}{3\Sigma^c}$ is the square of the diffusion length in the moderator. The solution to Equation (1.1) can be written

$$\Phi(\bar{r}) = 1 + \sum_k \sum_n \Phi_{n,k} \frac{K_n(|\bar{r} - \bar{r}_k|/L)}{K_n(R_k/L)} e^{in\varphi_k}, \tag{2.1}$$

where k is the block number, φ_k is the angle between a given direction and the vector $k(\bar{r}_k)$ from the center of block k to the point $\bar{r}$, and R_k is the radius of block k.

The coefficients $\Phi_{n,k}$ are determined from the conditions at the block boundaries. In order to include these conditions, let us consider this solution in the neighborhood of block number l:

$$\begin{aligned}\Phi_l(\bar{\rho}_l) = \Phi(\bar{r}_l + \bar{\rho}_l) = \\ = 1 + \sum_n \Phi_{n,l} \frac{K_n(\rho_l/L)}{K_n(R_l/L)} e^{in\varphi_l} + \\ + \sum_{k \neq l} \sum_n \Phi_{n,k} \frac{K_n(|\bar{r}_l - \bar{r}_k + \bar{\rho}_l|/L)}{K_n(R_k/L)} e^{in\varphi_k}, \\ \bar{\rho}_l = \bar{r} - \bar{r}_l.\end{aligned} \tag{3.1}$$

Let us expand this solution in a Fourier series in φ_l, using the expansion formula for the Bessel function

$$\begin{aligned}K_n(|\bar{r}_{l,k} + \bar{\rho}_l|/L) e^{in\varphi_k} = \\ = \sum_m I_m(\rho_l/L) e^{-im\varphi_l} \times \\ \times K_{m+n}(r_{l,k}/L) e^{-i(m+n)\vartheta_{k,l}} (-1)^m,\end{aligned} \tag{4.1}$$

where $\vartheta_{k,l}$ is the angle between the fixed direction and the vector $\bar{r}_{k,l} = \bar{r}_k - \bar{r}_l$. Inserting (4.1) into (3.1), we obtain

$$\begin{aligned}\Phi_l(\bar{\rho}_l) = 1 + \sum_n \Phi_{n,l} \frac{K_n(\rho_l/L)}{K_n(R_l/L)} e^{in\varphi_l} + \\ + \sum_m I_m(\rho_l/L) e^{-im\varphi_l} (-1)^m \times \\ \times \sum_{k \neq l} \sum_n \Phi_{n,k} \frac{K_{m+n}(r_{l,k}/L)}{K_n(R_k/L)} e^{-i(m+n)\vartheta_{k,l}}.\end{aligned} \tag{5.1}$$

Let us perform the following replacement of indices in the double sum over n and m:

$$m \to -n, \qquad n \to -m.$$

Noting that the functions I_n and K_n depend only on the absolute value of their index, we obtain the following Fourier series for $\Phi_l(\bar{\rho}_l)$:

$$\begin{aligned}\Phi_l(\bar{\rho}_l) = \sum_n e^{in\varphi_l} \left\{ \delta(n) + \Phi_{n,l} \frac{K_n(\rho_l/L)}{K_n(R_l/L)} + \right. \\ + (-1)^n I_n(\rho_l/L) \times \\ \left. \times \sum_{k \neq l} \sum_m \Phi_{-m,k} \frac{K_{m-n}(r_{l,k}/L)}{K_n(R_k/L)} e^{i(m+n)\vartheta_{k,l}} \right\}.\end{aligned} \tag{6.1}$$

If within the block the neutron field is described by the diffusion equation

$$(1 - L_l^2\Delta)\,\Phi_l = 0, \tag{7.1}$$

then this field can be written

$$\Phi_{ll} = \sum_n e^{in\varphi_l} I_n(\rho_l/L_{ll})\, C_{n,l}. \tag{8.1}$$

It follows from this that the value of the radial logarithmic derivative of the n-th Fourier component of the neutron field, evaluated at the boundary, is a constant of the block, and is independent of the field outside of it:

$$\left.\frac{d\ln(\Phi_{ll})_n}{d\rho_l}\right|_{\rho_l=R_l} = \frac{1}{L_{ll}}\ln' I_n(R_l/L_{ll}), \tag{9.1}$$

where

$$\ln' I_n(x) = \frac{d\ln I_n(x)}{dx}.$$

But from the continuity of Φ and the diffusion flux $\frac{\lambda^{tr}}{3}\cdot\frac{d\Phi}{d\rho}$, it follows that the internal and external (in the moderator) logarithmic derivatives are related by

$$\left.\frac{d\ln(\Phi_l)_n}{d\rho_l}\right|_{\rho_l=R_l} = \frac{\lambda^{tr}_{ll}}{\lambda^{tr}}\cdot\left.\frac{d\ln(\Phi_{ll})_n}{d\rho_l}\right|_{\rho_l=R_l},$$

or in other words

$$\left.\begin{aligned}\left.\frac{d(\Phi_l)_n}{d\rho_l}\right|_{\rho_l=R_l} &= \frac{(\Phi_l)_n|_{\rho_l=R_l}}{\gamma_{l,n}},\\ \frac{1}{\gamma_{l,n}} &= \frac{\lambda^{tr}_{ll}/\lambda^{tr}}{L_{ll}}\ln' I_n\left(\frac{R_l}{L_{ll}}\right).\end{aligned}\right\} \tag{10.1}$$

Equation (10.1) may be considered the boundary condition necessary for the solution of (1.1), so that by changing the expression for the coefficient $\gamma_{l,n}$ we may take account of deviations from diffusion theory within the block and in its vicinity. We shall therefore go on to a solution of the problem without using the explicit expressions for the coefficients $\gamma_{l,n}$. Equations (6.1) and (10.1) may be used to obtain a system of equations for $\Phi_{n,l}$, namely

$$\begin{aligned}&\delta(n) + \Phi_{n,l}\left[1 - \frac{\gamma_{l,n}}{L}\ln' K_n(R_l/L)\right] +\\ &+(-1)^n\left[1 - \frac{\gamma_{l,n}}{L}\ln' I_n(R_l/L)\right]\times\\ &\times I_n(R_l/L)\sum_{k\neq l}\sum_m \Phi_{-m,k}\frac{K_{m+n}(\rho_{l,k}/L)}{K_n(R_k/L)}\times\\ &\times e^{i(m+n)\vartheta_{k,l}} = 0.\end{aligned} \tag{11.1}$$

In these equations the index n takes on all values from $-\infty$ to $+\infty$, and the index l takes on as many values as there are dissimilar (with respect to properties and positions) rods in the cell. Setting $\Phi_{n,k} = 0$ for

$n > N$, i.e., "cutting off" this infinite set of equations, we can calculate $\Phi_{n,k}$ and then calculate the neutron absorption in an arbitrary rod. In fact, the neutron flux incident on the l-th rod is

$$Q_l = 2\pi R_l \frac{\lambda^{tr}}{3} \int_0^{2\pi} \left(\frac{d\Phi}{d\rho_l}\right)_{\rho_l = R_l} \times \times \frac{d\varphi_l}{2\pi} = 2\pi R_l \frac{\lambda^{tr}}{3} \cdot \frac{(\Phi_l)_0|_{\rho_l = R_l}}{\gamma_{l,0}}, \tag{12.1}$$

where

$$(\Phi_l)_0|_{\rho_l = R_l} = 1 + \Phi_{0,l} + + I_0(R_l/L) \sum_{k \neq l} \sum_m \Phi_{-m,k} \frac{K_m(r_{l,k}/L)}{K_0(R_k/L)} e^{im\vartheta_{k,l}}. \tag{13.1}$$

If the slowing-down area in a cell is V, then according to the normalization taken for Equation (1.1), the total number of neutrons absorbed in the cell is $\Sigma^c V$. Therefore, the fraction of neutrons absorbed in the l-th rod is given by

$$\Theta_l = \frac{Q_l}{\Sigma^c V}, \tag{14.1}$$

and the thermal utilization factor, according to Equations (12.1) and (14.1), is

$$\Theta = \sum_l \Theta_l = \frac{2\pi L^2}{V} \sum_l \frac{R_l}{\gamma_{l,0}} (\Phi_l)_0|_{\rho_l = R_l}, \tag{15.1}$$

where the sum is taken over all the blocks of the cell.

Equations (11.1), (13.1), and (15.1) give the solution to the problem in general form, and the coefficients $\gamma_{l,n}$ can be calculated by Equation (10.1); if, on the other hand, the neutron field within the blocks and in their vicinity is found other than by the diffusion approximation, the coefficients $\gamma_{l,n}$ can be suitably corrected (see Section 3).

2. Lattice of Similar Blocks ("Simple" Lattice)

If all the blocks are similar and equally spaced, i.e., each elementary cell contains only one block, then Equations (11.1) reduce to

$$\left.\begin{aligned} &\delta(n) + \left[1 - \frac{\gamma_n}{L} \ln' I_n\left(\frac{R}{L}\right)\right] \times \\ &\qquad \times (\alpha_n + \beta_n \Phi_n) = 0, \\ &\alpha_n = \frac{I_n(R/L)}{K_n(R/L)} \cdot (-1)^n \sum_m \Phi_{-m} S_{m+n}, \\ &S_n = \sum_{r_k \neq 0} K_n(r_k/L) e^{in\vartheta_k}, \\ &\beta_n = \frac{1 - \frac{\gamma_n}{L} \ln' K_n(R/L)}{1 - \frac{\gamma_n}{L} \ln' I_n(R/L)}. \end{aligned}\right\} \tag{1.2}$$

Instead of (13.1), we obtain

$$(\Phi)_0|_{\rho = R} = 1 + \Phi_0 + \alpha_0. \tag{2.2}$$

Let us introduce the notation

$$S_0' = \sum_m \frac{\Phi_{-m}}{\Phi_0} S_m. \tag{3.2}$$

Then Equations (1.2) and (3.2) give

$$1 + \left[1 - \frac{\gamma_0}{L} \ln' I_0\left(\frac{R}{L}\right)\right](\alpha_0 + \beta_0 \Phi_0) = 0,$$

where

$$\alpha_0 = \frac{I_0(R/L)}{K_0(R/L)} S_0' \Phi_0,$$

whence

$$\Phi_0 = \frac{-K_0}{K_0 + I_0 S_0' + \frac{\gamma_0}{L}(K_1 - I_1 S_0')} \tag{4.2}$$

(the argument R/L of the functions K_0, K_1, I_0, I_1 is omitted).

From Equations (2.2), (4.2), and (15.1) we obtain an expression for Θ:

$$\Theta = \frac{2\pi L^2}{V} \cdot \frac{R}{\gamma_0}\left[1 - \frac{K_0 + I_0 S_0'}{K_0 + I_0 S_0' + \frac{\gamma_0}{L}(K_1 - I_1 S_0')}\right],$$

or after some simple transformations

$$\frac{1-\Theta}{\Theta} = \frac{V\gamma_0}{2\pi R L^2} + \left(\frac{1-\Theta}{\Theta}\right)_{\text{bet}} \tag{5.2}$$

where

$$\left(\frac{1-\Theta}{\Theta}\right)_{\text{bet}} = \frac{V}{2\pi L^2} \cdot \frac{K_0 + I_0 S_0'}{\frac{R}{L}(K_1 - I_1 S_0')} - 1. \tag{6.2}$$

In Expression (6.2) the quantity S_0' is as yet undetermined. Equations (1.2) give (for $n \neq 0$)

$$\Phi_n = -\frac{\alpha_n}{\beta_n} = \frac{(-1)^{n+1}}{\beta_n} \cdot \frac{I_n}{K_n} \cdot \sum_m \Phi_m S_{n-m}$$

or

$$\left.\begin{aligned} X_n &= g_n \sum_m x_m S_{n-m}, \\ X_n &= \frac{\Phi_n}{\Phi_0}, \quad g_n = \frac{(-1)^{n+1}}{\beta_n} \cdot \frac{I_n}{K_n}, \end{aligned}\right\} \tag{7.2}$$

whence, according to the notation of (3.2),

$$S_0' = \sum_m X_m S_{-m}. \tag{8.2}$$

The coefficients g_n decrease rapidly as $|n|$ increases, since the first term in the series expansion of $\frac{I_n}{K_n}$ in powers of R/L is

$$\frac{I_n}{K_n} = -\frac{(R/L)^{2|n|}}{(|n|!\,2^{|n|})^2}\,2|n| + \dots \tag{9.2}$$

We can therefore write series of the following form:

$$\left.\begin{aligned}
X_1 &= g_1[S_1 + X_1S_0 + X_{-1}S_2 + \dots] = \\
&= g_1S_1 + g_1^2(S_1S_0 + S_{-1}S_2) + \dots, \\
X_{-1} &= g_1[S_{-1} + X_{-1}S_0 + X_1S_{-2} + \dots] = \\
&= g_1S_{-1} + g_1^2(S_{-1}S_0 + S_1S_{-2}) + \dots, \\
X_2 &= g_2[S_2 + \dots], \\
X_{-2} &= g_2[S_{-2} + \dots], \\
S_0' &= S_0 + 2g_1S_1S_{-1} + \\
&\quad + g_1^2(S_{-1}^2S_2 + S_1^2S_{-2} + 2S_1S_{-1}S_0) + \\
&\quad + 2g_2S_2S_{-2} + \dots
\end{aligned}\right\} \tag{10.2}$$

The terms omitted in (10.2) contain the radius of the block to the sixth power and higher. In actuality, symmetry considerations show that some of the sums S_n vanish, which usually causes the term following S_0 to give only a small correction to S_0.

Let us assume, for instance, that the lattice possesses central symmetry (the blocks are located at the vertices of a parallelogram). Then all the blocks with $k \neq 0$ can be divided into two sets k_1 and k_2 such that when the set k_1 is rotated through an angle π, the blocks in it become coincident with those of set k_2. Then

$$\begin{aligned}
S_n &= \sum_{k_1} K_n(r_k/L)[e^{in\theta_{k_1}} + e^{in(\theta_{k_1}+\pi)}] = \\
&= (1 + e^{in\pi}) \sum_{k_1} K_n e^{in\theta_{k_1}}
\end{aligned}$$

and all S_n with odd n vanish. If the blocks are located at the vertices of a hexagon, then the lattice is transformed into itself by a rotation of $\frac{2\pi}{3}$. We then obtain

$$S_n = (1 + e^{in\frac{2\pi}{3}} + e^{in\frac{4\pi}{3}}) \sum_{k_1}' K_n e^{in\theta_{k_1}}$$

and $S_n \neq 0$ only if n is divisible by three. For a square lattice, nonvanishing S_n have n divisible by 4; for a triangular lattice, n divisible by 6. If we take account of the lattice symmetry, then (7.2) and (8.2) may be replaced by

$$X_{np} = g_{np} \sum_m X_{mp} S_{(n-m)p}, \tag{11.2}$$

$$S_0' = \sum_m X_{mp} S_{-mp}, \tag{12.2}$$

where p is the order of symmetry of the lattice (p = 2, 3, 4, 6). Then (10.2) becomes

$$S_0' = S_0 + 2g_p S_p S_{-p} + g_1^2 (S_{-p}^2 S_{2p} + S_p^2 S_{-2p} + 2S_0 S_p S_{-p}) + 2g_{2p} S_{2p} S_{-2p} + \dots \quad (13.2)$$

The terms omitted from (13.2) contain the block radius to the power 6p, which means at least to the twelfth power. The lattice symmetry also determines the nonzero values of S_n to some extent. In fact, if the lattice has an axis of symmetry, it is convenient to consider the angles with respect to this axis. Then

$$S_n = \sum_{\vartheta_k = 0, \pi} K_n + \sum_{\pi > \vartheta_k > 0} K_n (e^{in\vartheta_k} + e^{-in\vartheta_k}) = \sum_{\vartheta_k = 0, \pi} K_n + \sum_{\pi > \vartheta_k > 0} K_n 2\cos n\vartheta_k,$$

from which it follows that $S_n = S_{-n}$. Thus, when p > 2 (hexagonal, square, and triangular lattices), as well as for a rectangular or a rhombic lattice, for which p = 2, we have

$$S_0' = S_0 + 2g_p S_p^2 + 2g_1^2 S_p^2 (S_0 + S_{2p}) + 2g_{2p} S_{2p}^2 + \dots \quad (14.2)$$

Let us now calculate the correction $S_0' - S_0$ which is related to the asymmetry of the field at the block surface. For large indices the function K_p decreases rapidly as its argument increases (for arguments that are not large, $K_p\left(\frac{r}{L}\right) \approx \frac{(p-1)!\,2^{p-1}}{(r/L)^p}$). Therefore

$$S_p \approx pK_p\left(\frac{a}{L}\right) \approx \frac{p!2^{p-1}}{(a/L)^p},$$

where a is the lattice spacing (the distance between the axes of closest blocks).

We thus obtain, with the expressions for S_p and g_p,

$$S_0' - S_0 \approx \frac{(-1)^{p+1}}{\beta_p}\left(\frac{R}{a}\right)^{2p} \cdot p. \quad (15.2)$$

The coefficient β_p is of order of magnitude unity. In fact, for large indices and small arguments

$$\ln' I_p \approx p. \quad \ln' K_p \approx -p.$$

Expressions (10.1) and (1.1) give

$$\frac{1}{T_p} \approx \frac{\lambda_0^{tr}/\lambda^{tr}}{L_0} \cdot p, \quad \beta_p \approx \frac{1 + \frac{T_p}{L} p}{1 - \frac{T_p}{L} p} \approx \frac{1 + \frac{L_0 \lambda^{tr}}{L \lambda_0^{tr}}}{1 - \frac{L_0 \lambda^{tr}}{L \lambda_0^{tr}}}, \quad (16.2)$$

and since usually $L_\delta \lambda^{tr}/L\lambda^{tr}_\delta \ll 1$, the coefficient $\beta_p \approx 1$. Since the lattice spacing cannot be smaller than two block radii, an evaluation of the correction from Equation (15.2) gives

$$|S'_\theta - S_\theta| \leq \frac{p}{2^{2p}}. \tag{17.2}$$

For a triangular lattice this correction is $1.5 \cdot 10^{-3}$, for a square one it is $1.5 \cdot 10^{-2}$, and for a hexagonal it is $4.7 \cdot 10^{-2}$. Since the denominator of (6.2) and I_θ are almost 1 for small values of R/L, a variation of S'_θ by an amount $\delta S'_\theta$ causes $\frac{1-\Theta}{\Theta}$ to vary by an amount

$$\delta\left(\frac{1-\Theta}{\Theta}\right) \approx \frac{V}{2\pi L^2}\delta S'_\theta \tag{18.2}$$

Thus the "filament-like" block method, in which each block is a source (or more accurately, a sink) of the symmetric field, gives a very good approximation even for a hexagonal lattice. It should be noted that the maximum error due to the use of this method is of the same order as that due to the use of the "equivalent cell" method. In fact comparison of the sums S_θ for various lattices with equal cell areas shows that they differ extremely slightly from each other and are almost equal to the ratio

$$S = \frac{K_1(R_e/L)}{I_1(R_e/L)}, \tag{19.2}$$

where R_e is the radius of the equivalent (with respect to area) circular cell. For a triangular lattice $S_{\theta\Delta}$ differs from S only in the third place, and the percent error involved in replacing $S_{\theta\Delta}$ by S_θ becomes greater than 1% only if $R_e/L > 3$. For a square lattice, when $R_e/L < 1$, we have

$$S_{\theta\square} - S \approx 0.01.$$

and for larger values of R_e/L, this difference decreases. For a hexagonal lattice, when $R_e/L < 0.4$,

$$S_{\theta\bigcirc} - S \approx 0.07$$

and then decreases. But if S'_θ is replaced by S from (19.2) in Equation (6.2), it becomes the well known formula for an equivalent cell [2]. Thus, the error due to use of the "equivalent cell" method is given by (18.2). For a square lattice with thin blocks, $\delta S \approx 0.01$, and for a hexagonal lattice with thin blocks $\delta S \approx 0.07$. For the case of thicknest possible blocks ($R = a/2$) the error increases in a hexagonal lattice and decreases by the same order in a square one.

Let us note that the approximation (19.2) for the sums S_θ can be easily explained. For this purpose, let us consider the function

$$z(\bar{r}) = \sum_k K_\theta(|\bar{r} - \bar{r}_k|/L),$$

which is periodic and therefore is maximal at the cell boundaries. If we deform these boundaries to circles, we can approximate the function $z(\bar{r})$ by a symmetric one which satisfies the same equation, has the same singularity, and is maximal at the boundary. This will obviously be a linear combination of the Bessel functions K_θ and I_θ:

$$z(\bar{r}) \approx K_\theta(|\bar{r} - \bar{r}_0|/L) + AI_\theta(|\bar{r} - \bar{r}_0|/L), \tag{20.2}$$

where the constant A is determined from the condition that the function is maximum when $|\bar{r} - \bar{r}_0| = R_e$:

$$A = \frac{K_1(R_e/L)}{I_1(R_e/L)}. \tag{21.2}$$

Equations (20.2) and (21.2) give

$$S_0 = [z(\bar{r}) - K_0(|\bar{r} - \bar{r}_0|/L)]_{\bar{r}=\bar{r}_0} \approx$$
$$\approx AI_0(|\bar{r} - \bar{r}_0|/L)|_{\bar{r}=\bar{r}_0} = \frac{K_1(R_e/L)}{I_1(R_e/L)}.$$

Let us note also that if we neglect the difference between S_0^* and S_0, the heterogeneous correction $\left(\frac{1-\Theta}{\Theta}\right)_{het}$ becomes practically proportional to $1/L^2$. Explicitly, up to extremely tight lattices and extremely small values of L/R_e (up to 0.5 with an accuracy of 5%), we obtain the formula

$$\left(\frac{1-\Theta}{\Theta}\right)_{het} \approx$$
$$\approx \frac{V}{4\pi L^2}\left[\frac{\ln q}{(1-1/q)^2} - \frac{3}{2}\cdot\frac{1-1/3q}{1-1/q}\right], \tag{22.2}$$
$$q = \left(\frac{R_e}{R}\right)^2.$$

This formula can be obtained either by expanding (6.2) in a series with $S_0^* = S$ or from the expression

$$\left(\frac{1-\Theta}{\Theta}\right)_{het} = \bar{\Phi} - \Phi_{bound} \tag{23.2}$$

where $\bar{\Phi} - \Phi_{bound}$ (the mean difference between the neutron flux in the moderator and the flux at the block boundaries) is determined from Equation (1.1), neglecting neutron absorption

$$-L^2\Delta\Phi = 1. \tag{24.2}$$

Expression (22.2) is useful to bear in mind in calculating the temperature effect.

3. Nondiffusion Corrections

Boundary Conditions (10.1) are valid only when the diffusion equation may be used within the block. If this may not be done, but the blocks are located far from each other, the flux Φ extrapolated according to the diffusion equation from the moderator to the block boundary should satisfy a more complicated set of conditions, namely

$$\frac{d(\Phi)_n}{d\rho}\Big|_{\rho=R} = \sum_m \frac{(\Phi)_m|_{\rho=R}}{\gamma_{n,m}}. \tag{1.3}$$

The coefficients $\gamma_{n,m}$ may in general be found from the solution to the kinetic equations in the block and in its vicinity. However, this is not necessary. Indeed, as has been shown above, it is not necessary even to know the coefficients $\gamma_{n,n} = \gamma_n$ accurately, since the coefficients β_n which depend on γ_n, are almost 1 and enter into small corrections. As for the coefficients $1/\gamma_{n,m}$ for $n \neq m$, they would seem to play even a less important role and are furthermore small, since they vanish in the diffusion approximation.

It is much more important to know the exact value of the constant γ_θ, since it enters as a factor of the fundamental term in the expression for $\frac{1-\Theta}{\Theta}$. Indeed, according to Equation (15.2)

$$\left(\frac{1-\Theta}{\Theta}\right)-\left(\frac{1-\Theta}{\Theta}\right)_{\text{het}}=\frac{V\gamma_\theta}{2\pi RL^2}. \tag{2.3}$$

The constant γ_θ is known if we know the average over the block of the flux Φ_δ and the value of Φ_{bound} extrapolated from the moderator to the block boundary. Indeed, the neutron flux incident on the block can be written either with the aid of the diffusion equation

$$Q=2\pi R\frac{d(\Phi)_\theta}{d\rho}\Big|_{r=R}\frac{\lambda^{tr}}{3}=2\pi R\frac{(\Phi)_\theta|_{\rho=R}}{\gamma_\theta}\frac{\lambda^{tr}}{3}, \tag{3.3}$$

or can be written as the number of neutrons absorbed in the block

$$Q=\pi R^2\Sigma_\delta^c\Phi_\delta. \tag{4.3}$$

Equating these two expressions, we obtain

$$\gamma_\theta=\frac{2}{3}\cdot\frac{\lambda^{tr}}{R\Sigma_\delta^c}F, \tag{5.3}$$

where $F=\frac{\Phi_{\text{bound}}}{\Phi_\delta}$ (F is the inhomogeneity factor of the neutron field). Inserting this expression into Equation (2.3), we obtain

$$\left(\frac{1-\Theta}{\Theta}\right)-\left(\frac{1-\Theta}{\Theta}\right)_{\text{het}}=\frac{V\Sigma^c}{\pi R^2\Sigma_\delta^c}F. \tag{6.3}$$

In the diffusion approximation

$$F=\frac{I_0(R/L_1)\,R/L_2}{2I_1(R/L_1)}=1+\frac{1}{8}\cdot\frac{R^2}{L_1^2}+\dots \tag{7.3}$$

A more accurate expression for F may be obtained, for instance, by expanding the neutron flux in terms of the directions Ω, i.e., by expansion of the function $f(\vec{r},\Omega)$ in a series of spherical harmonics [3].

Then the solution for Φ can be written in the form

$$\Phi=\Phi_{\text{diff}}+\delta\Phi, \tag{8.3}$$

where Φ_{diff} satisfies the diffusion equation, and $\delta\Phi$ is the nondiffusion correction (outside the block).

In order to introduce corrections to the diffusion calculation we must know, in addition to F or γ_θ, the value of the integral $\int_R^\infty \delta\Phi 2\pi r\,dr$. Then the actual neutron absorption in the moderator can be found by subtracting from $(1-\Theta/\Theta)_{\text{het}}$ the quantity

$$\delta\left(\frac{1-\Theta}{\Theta}\right)_{het}=\frac{\Sigma^a}{\pi R^2\Sigma_\delta^a\Phi_\delta}\int_R^\infty|\delta\Phi|\,2\pi r\,dr,\qquad \delta\Phi<0. \tag{9.3}$$

SUMMARY

If the diffusion equation with effective boundary conditions can be applied to the space between the blocks, the formula for Θ can be written

$$\frac{1-\theta}{\theta}=\left(\frac{1-\theta}{\theta}\right)_{homo}F+\left(\frac{1-\theta}{\theta}\right)_{het}-\frac{\int_R^\infty|\delta\Phi|\,2\pi r\,dr\Sigma^a}{\pi R^2\Phi_\delta\Sigma_\delta^a},\quad F=\frac{\Phi_{bound}}{\Phi_\delta},$$

where Φ_{bound} is the neutron flux extrapolated from the moderator to the block boundary, and $\delta\Phi=\Phi-\Phi_{diff}$ is the nondiffusion correction to the flux. In the diffusion approximation $\delta\Phi=0$, and F is calculated by Equations (7.3).

The quantity $\left(\frac{1-\Theta}{\Theta}\right)_{het}$ can be calculated by the "equivalent cell" method [2], where it is denoted by E-1, or one may use the simplified formula (22.2). The order of magnitude of the error involved in using the "equivalent cell" method is

$$\delta\left(\frac{1-\Theta}{\Theta}\right)\sim\frac{V}{2\pi L^2}\cdot\frac{p}{2^{2p}},$$

where p is the order of symmetry of the lattice, V is the moderator area in the cell, and L is the diffusion length in the moderator.

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NEUTRON DIFFUSION IN A PLANE URANIUM-WATER LATTICE

Ya. V. Shevelev

The diffusion lengths for a plane uranium-water lattice are calculated for thermal-neutron diffusion parallel ($L_\parallel$) and perpendicular ($L_\perp$) to the layers. The calculation of $L_\perp$ is carried out in the diffusion approximation but can be used without the introduction of nondiffusion corrections since it is correct in the limiting cases (thick and thin layers). The calculation of $L_\parallel$ is carried out both in the diffusion and nondiffusion approximation; in the latter case, however, it is assumed that the absorption is small. In the case of thick layers, the ratio $\frac{L_\parallel^2}{L_\perp^2}$ may reach 1.9. It is shown that if a uranium-aluminum alloy is used the anisotropy becomes still larger. As the layer thickness is reduced the anisotropy is reduced and finally vanishes.

INTRODUCTION

If ordinary water is used as a moderator in a heterogeneous reactor the diffusion of thermal neutrons may depend strongly on direction. In a lattice with thick lumps the neutron diffusion coefficient along the axis of a lump may be twice as great as it is in the direction perpendicular to the axis of the lump; corresponding differences will be found in the diffusion length. A calculation of neutron diffusion in a lattice with cylindrical lumps is difficult. A simpler problem is considered in the present work: the diffusion of neutrons in a plane lattice, parallel and perpendicular to the layers. We may note that the anisotropy in a cylindrical lattice should be somewhat smaller than that of a plane lattice.

First the problem is solved in the diffusion approximation, taking absorption into account [1]. Then the problem of longitudinal diffusion is solved exactly, neglecting absorption. As a result one obtains an interpolation formula for calculating both absorption and departures from diffusion theory.

1. Transverse Diffusion With Absorption

In Fig. 1 a lattice is shown consisting of alternate layers of two materials, 1 and 2, with different capture cross sections per unit volume (Σ_1^c, Σ_2^c) and different transport lengths ($\lambda_1^{tr} = \frac{1}{\Sigma_1^{tr}}$, $\lambda_2^{tr} = \frac{1}{\Sigma_2^{tr}}$). The thicknesses of the layers are δ_1 and δ_2. A plane neutron source is located at "minus infinity" and diffusion takes place along the x axis in the positive direction.

We will assume that the layer thickness is large but that the capture cross section is small ($\delta\Sigma^{tr} \gg 1$, $\Sigma^c/\Sigma^{tr} \ll 1$). Under these conditions the diffusion equation can be used:

$$\frac{d}{dx}\frac{1}{3\Sigma^{tr}}\frac{d\Phi}{dx} - \Sigma^c\Phi = 0, \tag{1.1}$$

where Φ is continuous.

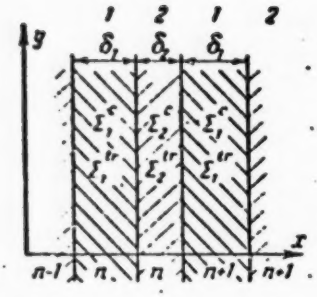

Fig. 1. Plane lattice of alternating layers of two materials (with neutron absorption).

The solution of this equation can be written in the following form:*

$$\left.\begin{aligned}\Phi_{1,n} &= e^{-\frac{\delta_1+\delta_2}{L_\perp}n}\left(A_1 \operatorname{ch}\frac{x}{L_1}+B_1 \operatorname{sh}\frac{x}{L_1}\right), \quad -\delta_1 < x < 0,\\ \Phi_{2,n} &= e^{-\frac{\delta_1+\delta_2}{L_\perp}n}\left(A_2 \operatorname{ch}\frac{x}{L_2}+B_2 \operatorname{sh}\frac{x}{L_2}\right), \quad 0 < x < \delta_2,\end{aligned}\right\} \tag{2.1}$$

where $L_\perp$ is the transverse diffusion length which is to be determined and

$$L_{1,2}=\frac{1}{\sqrt{3\Sigma^c_{1,2}\Sigma^{tr}_{1,2}}}$$

are the diffusion lengths in layers 1 and 2. The coefficients $A_{1,2}$ and $B_{1,2}$ are determined from the following boundary conditions:

$$\left.\begin{aligned}&\Phi_{1,n}|_{x=0}=\Phi_{2,n}|_{x=0},\\ &\frac{1}{\Sigma^{tr}_1}\frac{d\Phi_{1,n}}{dx}\Big|_{x=0}=\frac{1}{\Sigma^{tr}_2}\frac{d\Phi_{2,n}}{dx}\Big|_{x=0},\\ &\Phi_{1,n+1}|_{x=-\delta_1}=\Phi_{2,n}|_{x=\delta_2},\\ &\frac{1}{\Sigma^{tr}_1}\cdot\frac{d\Phi_{1,n+1}}{dx}\Big|_{x=-\delta_1}=\frac{1}{\Sigma^{tr}_2}\frac{d\Phi_{2,n}}{dx}\Big|_{x=\delta_2}\end{aligned}\right\} \tag{3.1}$$

The system of homogeneous equations which is obtained by substituting Equation (2.1) in the boundary Conditions (3.1) has a solution only for a definite value of $L_\perp$. This value can be determined by setting the determinant of the system equal to zero.

$$\begin{vmatrix} 1 & -1 & 0 & 0\\ 0 & 0 & \frac{1}{\Sigma^{tr}_1 L_1} & -\frac{1}{\Sigma^{tr}_2 L_2}\\ e^{-\frac{\delta_1+\delta_2}{L_\perp}}\cdot\operatorname{ch}\frac{\delta_1}{L_1} & \operatorname{ch}\frac{\delta_2}{L_2} & e^{-\frac{\delta_1+\delta_2}{L_\perp}}\cdot\operatorname{sh}\frac{\delta_1}{L_1} & \operatorname{sh}\frac{\delta_2}{L_2}\\ \frac{1}{\Sigma^{tr}_1 L_1}\cdot e^{-\frac{\delta_1+\delta_2}{L_\perp}}\cdot\operatorname{sh}\frac{\delta_1}{L_1} & \frac{1}{\Sigma^{tr}_2 L_2}\cdot\operatorname{sh}\frac{\delta_2}{L_2} & -\frac{1}{\Sigma^{tr}_1 L_1}\cdot e^{-\frac{\delta_1+\delta_2}{L_\perp}}\cdot\operatorname{ch}\frac{\delta_1}{L_1} & \frac{1}{\Sigma^{tr}_2 L_2}\cdot\operatorname{ch}\frac{\delta_2}{L_2}\end{vmatrix}=0. \tag{4.1}$$

Whence we find:

$$\operatorname{ch}\frac{\delta_1+\delta_2}{L_\perp}=\operatorname{ch}\left(\frac{\delta_1}{L_1}+\frac{\delta_2}{L_2}\right)+ +\frac{1}{2}\left(\sqrt{\frac{\Sigma^{tr}_2\Sigma^c_1}{\Sigma^{tr}_1\Sigma^c_2}}+\sqrt{\frac{\Sigma^{tr}_1\Sigma^c_2}{\Sigma^{tr}_2\Sigma^c_1}}-2\right)\times \times\operatorname{sh}\frac{\delta_1}{L_1}\cdot\operatorname{sh}\frac{\delta_2}{L_2}. \tag{5.1}$$

* [Here and in subsequent equations 'sh' = 'sinh' and 'ch' = 'cosh' – editor's note].

If the ratio of transport length to capture length is the same in both layers, the following simple result is obtained:

$$\left.\begin{aligned}\frac{\delta_1+\delta_2}{L_\perp}&=\frac{\delta_1}{L_1}+\frac{\delta_2}{L_2},\\ \frac{\Sigma_1^{tr}}{\Sigma_1^c}&=\frac{\Sigma_2^{tr}}{\Sigma_2^c}.\end{aligned}\right\} \tag{6.1}$$

If the thickness of the layer is small as compared with the diffusion length, the expression in (5.1) can be expanded in powers of $\frac{\delta}{L}$ and we obtain:

$$\left.\begin{aligned}L_\perp^2&\simeq\frac{1}{3(\Sigma^c)_{av}(\Sigma^{tr})_{av}},\\ (\Sigma^c)_{av}&=\frac{\delta_1\Sigma_1^c+\delta_2\Sigma_2^c}{\delta_1+\delta_2},\\ (\Sigma^{tr})_{av}&=\frac{\delta_1\Sigma_1^{tr}+\delta_2\Sigma_2^{tr}}{\delta_1+\delta_2},\end{aligned}\right\} \tag{7.1}$$

where $(\Sigma^c)_{av}$ and $(\Sigma^{tr})_{av}$ are the cross sections averaged over the lattice.

The expressions in (7.1) can be obtained another way if it is assumed that the neutron capture is small. Under these conditions, in the first approximation the neutron flux is determined by the equation

$$\frac{dq}{dx}=0,\ \text{where}\ q=\frac{1}{3\Sigma^{tr}}\cdot\frac{d\Phi}{dx}, \tag{8.1}$$

which may be considered as an analogue to the equation for the electric potential Φ in a conductor with a variable resistance $3\Sigma^{tr}$. The relation between the "current" q and the "potential" Φ in a system of series-connected conductors is given by the equation

$$q=\frac{\Phi_2-\Phi_1}{\int_1^2 3\Sigma^{tr}dx}, \tag{9.1}$$

in which the integral in the denominator plays the role of the "resistance" in the section 1-2. For a large number of layers one may introduce an average specific resistance $3(\Sigma^{tr})_{av}$ and then

$$q\simeq\frac{\Phi_2-\Phi_1}{3(\Sigma^{tr})_{av}\int_1^2 dx}=\frac{1}{3(\Sigma^{tr})_{av}}\left(\frac{d\Phi}{dx}\right)_{av} \tag{10.1}$$

If neutron equilibrium obtains with absorption taken into account,

$$q_2-q_1=\int_1^2\Phi\Sigma^c\,dx\simeq\Phi_{\ v}(\Sigma^c)_v\int_1^2 dx, \tag{11.1}$$

from Equations (10.1) and (11.1) one obtains a diffusion equation with absorption averaged over many layers

$$\Phi_{av}(\Sigma^c)_{av} \simeq \frac{dq}{dx}\,\delta_{av} \simeq \frac{1}{3(\Sigma^{tr})_{av}} \cdot \frac{d^2\Phi}{dx^2}\,\delta_{av}. \tag{12.1}$$

which is equivalent to Equation (7.1).

2. Longitudinal Diffusion With Absorption

We now consider the same medium but with a source located in the plane y = const (Fig. 1). In he diffusion equation

$$\nabla \frac{1}{3\Sigma^{tr}(x)} \nabla\Phi(x, y) - \Sigma^c(x)\,\Phi(x, y) = 0, \tag{1.2}$$

where Φ is continuous, the variables are separable. Taking

$$\Phi(x, y) = \Phi_x(x)\,\Phi_y(y), \tag{2.2}$$

we obtain:

$$3\Sigma^{tr}\left(\Sigma^c - \frac{\nabla \frac{1}{3\Sigma^{tr}} \nabla\Phi_x}{\Phi_x}\right) = \frac{\Delta\Phi_y}{\Phi_y}. \tag{3.2}$$

Whence it follows that

$$\frac{\Delta\Phi_y}{\Phi_y} = \text{const} = -\frac{1}{L_\parallel^2}, \tag{4.2}$$

where $L_\parallel$ is the diffusion length along the layer and

$$\frac{d}{dx}\frac{1}{3\Sigma^{tr}}\frac{d\Phi_x}{dx} + \frac{1}{3\Sigma^{tr}}\left(\frac{1}{L_\parallel^2} - \frac{1}{L^2}\right)\Phi_x = 0, \tag{5.2}$$

where Φ_x is continuous.

The quantity $L_\parallel$ is determined from the periodicity requirement on Φ_x. The solution of (5.2) can be written in the form

$$\left.\begin{aligned} &\Phi_{x1} = A_1 \operatorname{ch} p_1 x + B_1 \operatorname{sh} p_1 x \\ &\left(-\delta_1 < x < 0,\ p_1^2 = \frac{1}{L_\parallel^2} - \frac{1}{L_1^2}\right), \\ &\Phi_{x2} = A_2 \operatorname{ch} p_2 x + B_2 \operatorname{sh} p_2 x \\ &\left(0 < x < \delta_2,\ p_2^2 = \frac{1}{L_\parallel^2} - \frac{1}{L_2^2}\right). \end{aligned}\right\} \tag{6.2}$$

Taking into account the periodicity of Φ_x, the boundary conditions can be written in the following way:

$$\left.\begin{aligned} &\Phi_{x1}\big|_{x=0} = \Phi_{x2}\big|_{x=0}, \\ &\frac{1}{\Sigma_1^{tr}}\frac{d\Phi_{x1}}{dx}\bigg|_{x=0} = \frac{1}{\Sigma_2^{tr}}\frac{d\Phi_{x2}}{dx}\bigg|_{x=0}, \\ &\Phi_{x1}\big|_{x=-\delta_1} = \Phi_{x2}\big|_{x=\delta_2}, \\ &\frac{1}{\Sigma_1^{tr}}\frac{d\Phi_{x1}}{dx}\bigg|_{x=-\delta_1} = \frac{1}{\Sigma_2^{tr}}\frac{d\Phi_{x2}}{dx}\bigg|_{x=\delta_2}. \end{aligned}\right\} \tag{7.2}$$

Substituting (6.2) in (7.2) we obtain a homogeneous system of equations, the determinant of which is

$$\Delta = \begin{vmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & \frac{\mu_1}{\Sigma_1^{tr}} & -\frac{\mu_2}{\Sigma_2^{tr}} \\ -\operatorname{ch}\mu_1\delta_1 & \operatorname{ch}\mu_2\delta_2 & \operatorname{sh}\mu_1\delta_1 & \operatorname{sh}\mu_2\delta_2 \\ \frac{\mu_1}{\Sigma_1^{tr}}\operatorname{sh}\mu_1\delta_1 & \frac{\mu_2}{\Sigma_2^{tr}}\operatorname{sh}\mu_2\delta_2 & -\frac{\mu_1}{\Sigma_1^{tr}}\operatorname{ch}\mu_1\delta_1 & \frac{\mu_2}{\Sigma_2^{tr}}\operatorname{ch}\mu_2\delta_2 \end{vmatrix} \tag{8.2}$$

Setting the determinant equal to zero, we obtain an equation for $L_{\parallel}$:

$$1 = \operatorname{ch}(\mu_1\delta_1 + \mu_2\delta_2) + \\ + \frac{1}{2}\left(\frac{\mu_1\Sigma_2^{tr}}{\mu_2\Sigma_1^{tr}} + \frac{\mu_2\Sigma_1^{tr}}{\mu_1\Sigma_2^{tr}} - 2\right)\operatorname{sh}\mu_1\delta_1\operatorname{sh}\mu_2\delta_2, \\ \left(\mu_1^2 = \frac{1}{L_{\parallel}^2} - \frac{1}{L_1^2}\right). \tag{9.2}$$

If the absorption is weak and the diffusion lengths L_1 and L_2 do not differ greatly, the following substitutions may be used in Equation (9.2)

$$\operatorname{sh}\mu_1\delta_1 \cdot \operatorname{sh}\mu_2\delta_2 \simeq \mu_1\mu_2\delta_1\delta_2,$$
$$\operatorname{ch}(\mu_1\delta_1 + \mu_2\delta_2) \simeq 1 + \frac{1}{2}(\mu_1\delta_1 + \mu_2\delta_2)^2.$$

Then the following formula is obtained for $L_{\parallel}$

$$L_{\parallel}^2 \simeq \frac{(\lambda^{tr})_{av}}{3(\Sigma^c)_{av}}, \quad (\lambda^{tr})_{av} = \frac{\delta_1\lambda_1^{tr} + \delta_2\lambda_2^{tr}}{\delta_1 + \delta_2}. \tag{10.2}$$

where $\frac{(\lambda^{tr})_{av}}{3}$ is the average diffusion coefficient.

In Equation (7.1) the diffusion "resistance" $3\Sigma^{tr}$ is averaged similarly in (10.2) we can average the diffusion "conductivity" $\frac{\lambda^{tr}}{3}$. This is completely reasonable since in contrast to the preceding case, in place of series-connected resistances we have parallel-connected resistances and, as is well-known, the "conductivities" are added.

It should be noted that in calculating L sometimes use is made of averaging formulas with a "weight factor" $\bar{\Phi}_i$ (the neutron fluxes in the different materials). This has a definite meaning: averaging Σ^{tr} gives $L_{\perp}$, while averaging λ^{tr} gives $L_{\parallel}$. However, the values of $\bar{\Phi}_i$ cannot be identified with those which determine the thermal-neutron utilization factor. This is obvious from a comparison of Equation (5.2) with the equation for the neutron flux in the presence of a uniform field of epithermal neutrons:

$$\frac{d}{dz}\frac{1}{3\Sigma^{tr}}\frac{d\Phi}{dz} + \frac{1}{3\Sigma^{tr}}\left(-\frac{1}{L^2}\right)\Phi + \begin{cases} 0 & \text{(in the uranium),} \\ \text{const} & \text{(in the water)} \end{cases} = 0. \tag{11.2}$$

Equations (5.2) and (11.2) are different both in the lump and in the moderator.

Equations (7.1) and (10.2) indicate that a lattice in which the volumes of the lumps and moderator are almost the same but in which there is an essential difference in transport length, there should be observed a strong anisotropy:

$$\left.\begin{aligned}\frac{L_{\parallel}^2}{L_{\perp}^2} \simeq \frac{\delta_1/\Sigma_1^{tr} + \delta_2/\Sigma_2^{tr}}{\delta_1+\delta_2} \cdot \frac{\delta_1\Sigma_1^{tr} + \delta_2\Sigma_2^{tr}}{\delta_1+\delta_2} = \\ = 1 + \frac{\lambda_1^{tr}/\lambda_2^{tr} + \lambda_2^{tr}/\lambda_1^{tr} - 2}{(\sqrt{\delta_1/\delta_2} + \sqrt{\delta_2/\delta_1})^2}.\end{aligned}\right\} \qquad (12.2)$$

For example, in cold water ($\lambda_{tr_{H_2O}} = 0.48$ cm) and in uranium ($\lambda_U^{tr} = 2.6$ cm) and for equal volumes ($\delta_1 = \delta_2$); from Equation (12.2) it follows that $\frac{L_{\parallel}^2}{L_{\perp}^2} = 1.90$. The anisotropy will be still greater if the uranium is replaced by a uranium-aluminum alloy since the transport length in aluminum is 5 times larger than it is in uranium. However, in this case the diffusion approximation and Equation (10.2) are obviously no longer useful. The point being that if the transport lengths are large compared with the thickness of the layer the medium may be considered as a homogeneous nuclear mixture. Then, regardless of the direction of diffusion, Equation (7.1) applies and the average cross sections are used.

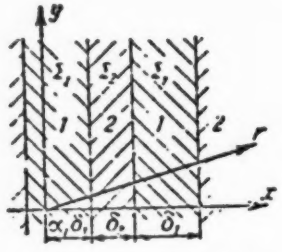

Fig. 2. Plane lattice with alternating layers of two materials (without neutron absorption).

In order to determine the limits of applicability of the diffusion theory, we now solve exactly the problem of neutron diffusion in plane layers; for simplicity we consider the case in which there is no absorption and in which the scattering is spherically symmetric (Fig. 2).

3. Exact Solution for Longitudinal Diffusion Without Absorption

The small asymmetry which occurs in scattering in cold water, will be taken into account layer, by replacing the mean-free path $\lambda = \frac{1}{\Sigma}$ by the transport length λ^{tr}.

If the source is distributed over the plane y = const the neutron flux depends linearly on y

$$\Phi = \Phi_0 + \frac{d\Phi}{dy} \cdot y, \qquad (1.3)$$

where Φ_0 and $\frac{d\Phi}{dy}$ are constants. This is apparent in two limiting cases: in the diffusion approximation (thick layers) and in the "homogeneous" approximation (thin layers). In the general case this is easily shown, starting from the equation given by Peierls [2],

$$\Phi(\bar{r}_0) = \int d\bar{r}\,\Sigma(\bar{r}) \frac{\Phi(\bar{r})}{4\pi|\bar{r}-\bar{r}_0|^2} e^{-\int_{\bar{r}_0}^{\bar{r}} \Sigma(\bar{r}')\,dr'}, \qquad (2.3)$$

where the integral in the exponential is taken along the line connecting the points $\bar{r}$ and $\bar{r}_0$ while the integral over $d\bar{r}$ is taken over all space.

If the constant Φ_0 is substituted in the right side of the Equation (2.3), then as a result of the integration we obtain Φ_0 since the constant undoubtedly appears as a solution for Equation (2.3). If, however, in the right

side of this equation we substitute the antisymmetric function $\frac{d\Phi}{dy}\,y$ and if the point $\bar{r}_0$ lies in the plane $y = 0$, then we obtain a vanishing result (the integrals over the upper and lower semiplanes differ only in sign). Thus, Equation (1.3) satisfies Equation (2.3).

Knowing $\Phi(\bar{r})$ it is then possible to compute the increment in the neutron flux which intersects the plane $y = 0$ from above:

$$\int q\,dx = \int dx \int d\bar{r}\,\Sigma(\bar{r})\,\frac{\Phi(\bar{r})}{4\pi|\bar{r}-\bar{r}_0|^2}\cos\vartheta\, e^{-\int_{\bar{r}_0}^{\bar{r}}\Sigma(\bar{r}')\,dr'}. \tag{3.3}$$

where ϑ is the angle between the y axis and the direction of a line from $\bar{r}_0$ to $\bar{r}$ (the point $\bar{r}_0$ lies in the plane $y = 0$). It is apparent that when we substitute a constant in the right side of Equation (3.3) in place of Φ we obtain a vanishing result. Hence,

$$\int q\,dx = \frac{d\Phi}{dy}\int dx \int \frac{\cos^2\vartheta}{4\pi|\bar{r}-\bar{r}_0|}\,\Sigma(\bar{r})\,e^{-\int_{\bar{r}_0}^{\bar{r}}\Sigma\,dr'}\,dr. \tag{4.3}$$

Replacing the volume element $d\bar{r}$ by $|\bar{r}-\bar{r}_0|^2\,d\Omega$, where $d\Omega$ is an element of solid angle, we obtain

$$\int q\,dx = \frac{d\Phi}{dy}\int dx \oint \frac{\cos^2\vartheta\,d\Omega}{4\pi} \times \\ \times \int_0^{\infty}\Sigma(\Omega, r)\,e^{-\int_0^{r}\Sigma(\Omega, r')\,dr'}\,r\,dr. \tag{5.3}$$

We now compute the integral

$$I(x, \Omega) = \int_0^{\infty}\Sigma(\Omega, r)\,e^{-\int_0^{r}\Sigma(\Omega, r')\,dr'}\,r\,dr. \tag{6.3}$$

We will call the quantity $\mu_{1,2}(\Omega)$ the "absorption factor" of the layer 1, 2 in the direction Ω, that is, the product of the cross section $\Sigma_{1,2}$ and the length of the ray Ω in layer 1, 2. Let $\alpha_1\delta_1$ be the distance from the point $\underline{x}$ to the closest boundary, intersected by the ray Ω (Fig. 2). Then

$$I(x, \Omega) = I(\alpha_1, \mu_1, \mu_2) = \\ = \int_0^{\alpha_1\mu_1} e^{-\mu}\frac{\mu}{\Sigma_1}\,d\mu + \int_0^{\mu_2} e^{-\alpha_1\mu_1-\mu}\left(\frac{\alpha_1\mu_1}{\Sigma_1}+\frac{\mu}{\Sigma_2}\right)d\mu + \\ + \int_0^{\mu_1} e^{-\alpha_1\mu_1-\mu_2-\mu}\left(\frac{\alpha_1\mu_1}{\Sigma_1}+\frac{\mu_2}{\Sigma_2}+\frac{\mu}{\Sigma_1}\right)d\mu + \\ + \int_0^{\mu_2} e^{-\alpha_1\mu_1-\mu_2-\mu_1-\mu} \times \\ \times\left(\frac{\alpha_1\mu_1}{\Sigma_1}+\frac{\mu_2}{\Sigma_2}+\frac{\mu_1}{\Sigma_2}+\frac{\mu}{\Sigma_2}\right)d\mu + \ldots \tag{7.3}$$

Computing the integral in (6.3) using Equation (7.3), we arrive at the following result:

$$I(x_1, \mu_1, \mu_2) = \frac{1}{\Sigma_1} + e^{-x_1/\mu_1} \cdot \frac{1 - e^{-\tau_2}}{1 - e^{-\tau_1 - \tau_2}} \cdot \left(\frac{1}{\Sigma_2} - \frac{1}{\Sigma_1}\right). \tag{8.3}$$

Now in Equation (5.3) it is possible to perform the integration with respect to $\underline{x}$:

$$\int I(x, \Omega)\, dx = \int dx \cdot \frac{\int_0^{a_1} I_1\, dx + \int_0^{a_2} I_2\, dx}{\delta_1 + \delta_2} = \int dx \cdot \left[\frac{\delta_1 \int_0^1 I_1(x_1, \mu_1, \mu_2)\, dx_1}{\delta_1 + \delta_2} + \frac{\delta_2 \int_0^1 I_2(x_2, \mu_2, \mu_1)\, dx_2}{\delta_1 + \delta_2}\right]. \tag{9.3}$$

Integrating Equation (8.3) with respect to α and substituting the result in (9.3), we have:

$$\int I(x, \Omega)\, dx = \int dx \cdot \left\{\frac{\delta_1/\Sigma_1 + \delta_2/\Sigma_2}{\delta_1 + \delta_2} + \frac{(1/\Sigma_2 - 1/\Sigma_1)}{\delta_1 + \delta_2}\left(\frac{\delta_1}{\mu_1} - \frac{\delta_2}{\mu_2}\right)\frac{(1 - e^{-\tau_1})(1 - e^{-\tau_2})}{1 - e^{-\tau_1 - \tau_2}}\right\}. \tag{10.3}$$

From Equations (10.3) and (5.3) we obtain an expression for the directed neutron flux averaged over the plane $y = 0$:

$$q_{av} = \frac{d\Phi}{dy} = \oint \frac{\cos^2\vartheta\, d\Omega}{4\pi} \left\{\frac{\delta_1\lambda_1 + \delta_2\lambda_2}{\delta_1 + \delta_2} - \frac{(\lambda_2 - \lambda_1)^2}{\delta_1 + \delta_2}\left[1 - \frac{e^{-\tau_1} + e^{-\tau_2} - 2e^{-\tau_1 - \tau_2}}{1 - e^{-\tau_1 - \tau_2}}\right] u\right\}, \tag{11.3}$$

where $u(\Omega)$ is the ratio of the thickness of a layer to the length of the ray Ω in this layer. The element of solid angle $d\Omega$ can be expressed in terms of the parameter $\underline{u}$ and the angle ϑ:

$$d\Omega = \frac{|du|\,|dv|}{\sqrt{1 - u^2 - v^2}}, \quad v = \cos\vartheta, \quad u^2 + v^2 < 1. \tag{12.3}$$

Using this result, Equation (11.3) can be written in the form

$$q_{av} = \frac{d\Phi}{dy} = 8\int_0^1 du \int_0^{\sqrt{1-u^2}} dv\, \frac{v^2}{4\pi} \frac{1}{\sqrt{1 - u^2 - v^2}} \times \times \left\{\lambda_{av} - \frac{(\lambda_2 - \lambda_1)^2}{\delta_1 + \delta_2}\left[1 - f\left(\frac{\delta_1/\Lambda_1}{u}, \frac{\delta_2/\Lambda_2}{u}\right)\right] u\right\}, \tag{13.3}$$

where

$$f(\mu_1, \mu_2) = \frac{e^{-\mu_1} + e^{-\mu_2} - 2e^{-\mu_1-\mu_2}}{1-e^{-\mu_1-\mu_2}} \tag{14.3}$$

and λ_{av} is determined through λ_1 and λ_2 in accordance with Equation (10.2). Integrating with respect to y, we obtain

$$q_{av} = \frac{d\Phi}{dy}\int_0^1 \frac{1-u^2}{2}\left\{\lambda_{av} - \frac{(\lambda_1-\lambda_2)^2}{\delta_1+\delta_2}\times\right.$$
$$\left.\times\left[1-f\left(\frac{\delta_1/\lambda_1}{u}, \frac{\delta_2/\lambda_2}{u}\right)\right]u\right\}du,$$

and then

$$q_{cp} = \frac{d\Phi}{dy}\cdot\frac{\lambda_{eff}}{3}, \tag{15.3}$$

where

$$\lambda_{eff} = \lambda_{av}\left[1 - \frac{3}{8}\frac{(\lambda_1-\lambda_2)^2}{\lambda_1\delta_1+\lambda_2\delta_2}(1-\bar{f})\right], \tag{16.3}$$

$$\bar{f}\left(\frac{\delta_1}{\lambda_1}, \frac{\delta_2}{\lambda_2}\right) = 4\int_0^1 (1-u^2)\,u\,f\left(\frac{\delta_1/\lambda_1}{u}, \frac{\delta_2/\lambda_2}{u}\right)du. \tag{17.3}$$

The exponential correction in $\bar{f}$ takes into account the "through flights" of neutrons across the layer. It may be computed by expanding $f(\mu_1, \mu_2)$ in a series:

$$\bar{f}(\mu_1, \mu_2) = \sum_{n=0}^{\infty}\varphi[n\mu_1+(n+1)\mu_2] + \varphi[(n+1)\mu_1+n\mu_2] - 2\varphi[(n+1)\mu_1+(n+1)\mu_2]. \tag{18.3}$$

where

$$\left.\begin{aligned}\varphi(\mu) &= 4\int_0^1 (1-u^2)\,ue^{-\mu/u}\,du = \\ &= e^{-\mu}\left(1-\frac{5}{2}\mu-\frac{1}{6}\mu^2+\frac{1}{6}\mu^3\right) + \\ &\quad + \left(2\mu^2-\frac{1}{6}\mu^4\right)\int_{\mu}^{\infty} e^{-t}\cdot\frac{dt}{t}.\end{aligned}\right\} \tag{19.3}$$

The function $\varphi(\mu)$ can be expressed in terms of the following expansions for large and small values of the argument:

$$\varphi(\mu)=\begin{cases} e^{-\mu}\cdot\left(\frac{8}{\mu^2}+\dots\right) & \text{for } \mu\to\infty, \\ 1-\frac{8}{3}\mu+2\mu^2\ln\frac{1}{\mu}+\dots & \text{for } \mu\to 0. \end{cases} \qquad (20.3)$$

If $\mu_2 = \infty$, then $\bar{f}(\mu_1, \mu_2) = \varphi(\mu_1)$. The values of the function $\bar{f}$ are given in the table.

Values of the Function $\bar{f}(\mu_1, \mu_2)$

μ_2 \ μ_1	0	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.5	2.0	3.0	4.0	∞
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.000	0.874	0.838	0.821	0.810	0.800	0.796	0.792	0.791	0.790	0.790	0.790	0.790
0.2	1.000	0.838	0.765	0.725	0.701	0.672	0.659	0.653	0.644	0.641	0.640	0.640	0.640
0.3	1.000	0.821	0.725	0.671	0.634	0.587	0.564	0.549	0.535	0.529	0.526	0.525	0.525
0.4	1.000	0.810	0.701	0.634	0.582	0.524	0.493	0.476	0.450	0.441	0.436	0.435	0.434
0.6	1.000	0.800	0.672	0.587	0.524	0.442	0.400	0.387	0.332	0.316	0.307	0.306	0.305
0.8	1.000	0.796	0.659	0.564	0.493	0.400	0.340	0.299	0.256	0.233	0.220	0.218	0.217
1.0	1.000	0.792	0.653	0.549	0.476	0.387	0.299	0.256	0.202	0.177	0.161	0.159	0.157
1.5	1.000	0.791	0.644	0.535	0.450	0.332	0.256	0.202	0.129	0.100	0.083	0.075	0.074
2.0	1.000	0.790	0.641	0.529	0.441	0.316	0.233	0.177	0.100	0.066	0.044	0.038	0.035
3.0	1.000	0.790	0.640	0.526	0.436	0.307	0.220	0.161	0.083	0.044	0.018	0.011	0.009
4.0	1.000	0.790	0.640	0.525	0.435	0.306	0.218	0.159	0.075	0.038	0.011	0.005	0.002
∞	1.000	0.790	0.640	0.525	0.434	0.305	0.217	0.157	0.074	0.035	0.009	0.002	0.000

4. Interpolation Formulas

In the transverse diffusion case the departures from diffusion theory should be small because the diffusion Equation (5.1) gives correct results for vanishingly thin layers. The following interpolation formula is given for longitudinal diffusion:

$$L_{\parallel}^2 \simeq (L_{\parallel}^2)_{\text{diff}} \cdot \frac{\lambda_{\text{eff}}}{\lambda_{\text{av}}}, \qquad (1.4)$$

where $(L_{\parallel}^2)_{\text{diff}}$ is determined from Equation (9.2) while $\frac{\lambda_{\text{eff}}}{\lambda_{\text{av}}}$ is determined from Equation (16.3). If the absorption is small, one can use the expansions in (5.1) and (9.2). Then,

$$\frac{L_{\parallel}^2}{L_{\perp}^2} \simeq \left[1+\frac{\lambda_1^{tr}/\lambda_2^{tr}+\lambda_2^{tr}/\lambda_1^{tr}-2}{\left(\sqrt{\delta_1/\delta_2}+\sqrt{\delta_2/\delta_1}\right)^2}\right]\times \times\left[1-\frac{3}{8}\frac{(\lambda_1^{tr}-\lambda_2^{tr})^2}{\lambda_1^{tr}\delta_2+\lambda_2^{tr}\delta_2}\left\langle 1-\bar{f}\left(\frac{\delta_1}{\lambda_1^{tr}},\frac{\delta_2}{\lambda_2^{tr}}\right)\right\rangle\right]. \qquad (2.4)$$

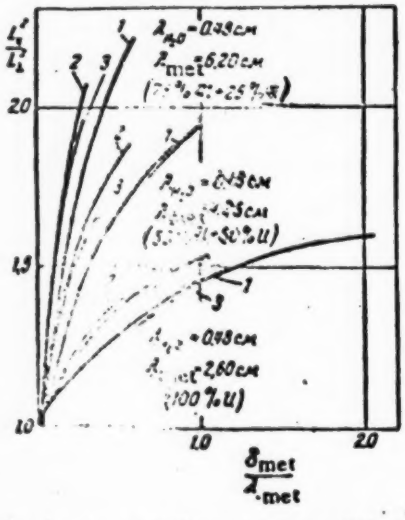

Fig. 3. [illegible] in thermal neutron-diffusion in a layered [illegible] water-uranium-aluminum alloy.

[illegible] 1: $\frac{\delta_{H_2O}}{\delta_{met}}$ = 1.0: $\frac{\delta_{H_2O}}{\delta_{met}}$ = 2.0.

In Fig. 3 are shown curves obtained from Equation (2.4) which indicate the dependence of the anisotropy on the thickness of the metal layer

δ_4 (a uranium-aluminum alloy). The parameters for the curves are the ratio of the volume of water to the volume of metal $\frac{\delta_2}{\delta_1}$ and the ratio of the volume of aluminum to the volume of uranium (x). Using these the following values of the constants were found:

$$\lambda_{H_2O}^{tr} = 0.48 \text{ cm},$$
$$\lambda_{U}^{tr} = 2.6 \text{ cm},$$
$$\lambda_{Al}^{tr} = 11.5 \text{ cm},$$
$$\lambda_{Al+U}^{tr} = \frac{1+x}{1/\lambda_{U}^{tr} + x/\lambda_{Al}^{tr}}.$$

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A VARIATIONAL METHOD FOR HOMOGENIZING A HETEROGENEOUS MEDIUM

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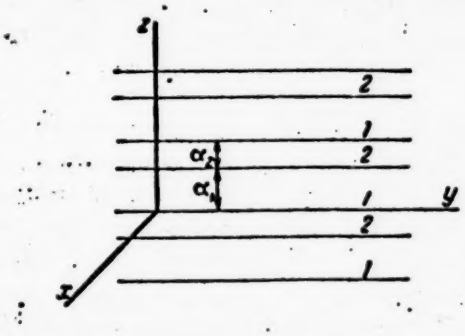
A heterogeneous medium is homogenized by solving the variational problem corresponding to the kinetic diffusion equation for monoenergetic neutrons. It is established that the homogenized constants of a heterogeneous medium depend in general on the mutual orientation of the neutron flux density and anisotropy of the heterogeneous medium. As the thickness of the layers which make up the heterogeneous medium approaches zero, the values of the homogenized constants begin not to depend on the above mutual orientation and become equal to the constants of the corresponding homogeneous mixture. This last conclusion is in contradiction to that of Spinrad [3]. However, since in this article the results are obtained by using the kinetic theory (Spinrad [3] used elementary neutron diffusion theory), we may assert that they are more nearly correct.

INTRODUCTION

In determining the properties of a heterogeneous reactor or in evaluating some experiments on neutron diffusion it becomes necessary to deal with the problem of homogenizing a heterogeneous medium. By this we mean that it becomes necessary to find the average values of the nuclear constants of the heterogeneous medium, so that these can be used, although approximately, to replace the neutron flux distribution function in the heterogeneous medium by the corresponding distribution function in the "homogeneous" medium. An example of this is the homogenization of the lattice of a heterogeneous reactor for which the fuel channels and moderator are replaced by an effective homogeneous medium. This problem, however, is also of theoretical interest.



Plane lattice of alternating layers of two media 1 and 2.

The present article is devoted to the homogenization of the heterogeneous medium shown in the figure. Infinite flat plates of two different substances 1 and 2, whose thicknesses are α_1 and α_2 ($\alpha = \alpha_1 + \alpha_2$), are alternately placed; the coordinate system is chosen so that the z axis is vertical to the planes of the lamina. For simplicity let us assume that neutron scattering in mediums 1 and 2 is isotropic, and that neither medium contains fissionable matter. This means that media 1 and 2 may be characterized by macroscopic neutron scattering (Σ_s) and absorption (Σ_a) cross sections with corresponding indices 1 and 2 for each medium ($\Sigma = \Sigma_s + \Sigma_a$). Let us assume further that the neutron sources are at infinity and emit monoenergetic (thermal) neutrons.

The kinetic equation for the neutron distribution function $\phi_0(r)$ can be written [1]

$$\phi_0(r) = \int\int\int e^{-\rho} d\rho \cdot \frac{\Sigma_s(r')}{\Sigma(r')} \cdot \phi_0(r') \cdot \frac{1}{4\pi} \cdot d\varphi \cdot d\vartheta. \tag{1}$$

Here the distance $\rho = |r - r'|$ is measured in units of the mean free path Σ^{-1} of the neutrons in the region spanned by ρ. The variables φ and ϑ give the direction of the vector $r - r'$ at the point r, where ϑ is the cosine of the polar angle ($\rho\vartheta = z - z'$), and φ is the azimuthal angle of the vector $r - r'$ at the point r.

We shall consider two separate cases:

1. A plane neutron source parallel to the z = const planes and located at infinity. Then the neutron flux $\phi_0(r)$ is a function only of z. Integrating over φ and transforming the double integral ($\rho\vartheta = z - z'$), Equation (1) may be obtained in the following form:

for medium 1

$$\begin{aligned}\phi_0(z + k\alpha) = \sum_{k'=-\infty}^{\infty} \int_0^{\alpha_1} dz' E_1(|\Sigma_1(z - z') + \\ + (k - k')(\Sigma_1\alpha_1 + \Sigma_2\alpha_2)|) \frac{\Sigma_{s1}}{2} \phi_0(z' + k'\alpha) + \\ + \sum_{k'=-\infty}^{\infty} \int_0^{\alpha_2} dz' E_1(|\Sigma_1 z - \Sigma_2 z' + \\ + (k - k')(\Sigma_1\alpha_1 + \Sigma_2\alpha_2) - \Sigma_1\alpha_1|) \times \\ \times \frac{\Sigma_{s2}}{2} \phi_0(z' + k'\alpha + \alpha_1),\end{aligned} \tag{2a}$$

for medium 2

$$\begin{aligned}\phi_0(z + k\alpha + \alpha_1) = \sum_{k'=-\infty}^{\infty} \int_0^{\alpha_1} dz' E_1(|\Sigma_2 z - \Sigma_1 z' + \\ + (k - k')(\Sigma_1\alpha_1 + \Sigma_2\alpha_2) + \Sigma_1\alpha_1|) \times \\ \times \frac{\Sigma_{s1}}{2} \phi_0(z' + k'\alpha) + \sum_{k'=-\infty}^{\infty} \int_0^{\alpha_2} dz' E_1(|\Sigma_2(z - z') + \\ + (k - k')(\Sigma_1\alpha_1 + \Sigma_2\alpha_2)|) \times \\ \times \frac{\Sigma_{s2}}{2} \phi_0(z' + k'\alpha + \alpha_1),\end{aligned} \tag{2b}$$

where the point $z + k\alpha$ is in medium 1, and $z + \alpha_1 + k\alpha$ is in medium 2.

We are here using relative coordinates z, z' so that in medium 1, $0 \le z, z' \le \alpha_1$, and in medium 2, $0 \le z, z' \le \alpha_2$. The integers k and k' give the ordinal number of the cell in the linear lattice (see the figure). The function $E_n(z)$ is defined as in neutron diffusion theory by the expression

$$E_n(z) = \int_1^{\infty} dt \cdot t^{-n} \cdot e^{-zt}. \tag{3}$$

Such a positioning of the source and medium leads to a definite value for the "diffusion length" if we use this expression in analogy with a homogeneous medium. This diffusion length depends on the mutual orientation of the neutron flux density and the lattice. In the given case, when the neutrons are incident vertically on the layers of media 1 and 2, we shall denote the reciprocal of the diffusion length by $\varkappa_{\perp}$.

2. A plane neutron source parallel to the y-z (x = const) plane and located at infinity. The neutrons from such a source are incident parallel to the layers of media 1 and 2; they give a different value for the diffusion length of the heterogeneous medium, whose reciprocal we shall denote by $\varkappa_{\parallel}$. This is the second interesting case of the possible incident neutrons' directions.

In this case the neutron flux $\psi_0(x)$ can be written as a produce $\psi_0(x) = e^{\varkappa_{\parallel} x}$. The function $\psi(z)$ is a function only of the coordinate z, and the exponential $e^{\varkappa_{\parallel} x}$ is due to the plane neutron source at $x = \infty$ and corresponds to the asymptotic solution in the case of a homogeneous medium with a similar source. If we make use of the relations

$$(\rho \cdot \sqrt{1-\vartheta^2} \cdot \cos\varphi = x - x')$$

and

$$\int_0^{2\pi} e^{\varkappa_{\parallel}\rho\sqrt{1-\vartheta^2}\cos\varphi} \cdot d\varphi = 2\pi I_0\left(\varkappa_{\parallel}\rho\sqrt{1-\vartheta^2}\right) \text{ [2]},$$

Equation (1) can be written in the form

$$\psi(z) = \int\int e^{-\rho} d\rho \cdot \frac{\Sigma_a(z')}{\Sigma(z')} \psi(z') \times \times I_0\left(\varkappa_{\perp}\rho\sqrt{1-\vartheta^2}\right) \cdot \frac{1}{2} d\vartheta, \tag{4}$$

where $I_0(t)$ is a modified Bessel function of the first kind and of zero order with an imaginary argument.

By further transformation of the double integral ($\rho\ \vartheta = z - z'$) we may put Equation (4) into the form

$$\begin{aligned}
\psi(z + ka) = &\sum_{k'=-\infty}^{\infty} \int_0^{a_1} dz' \int_1^{\infty} \frac{dt}{t} \times \\
&\times \exp\{-|\Sigma_1(z-z') + (k-k')(\Sigma_1 a_1 + \\
&+ \Sigma_2 a_2)|t\} I_0(\varkappa_{\parallel}|z-z' + \\
&+ (k-k')a|\sqrt{t^2-1}) \frac{\Sigma_{a1}}{2} \psi(z' + k'a) + \\
&+ \sum_{k'=-\infty}^{\infty} \int_0^{a_2} dz' \int_1^{\infty} \frac{dt}{t} \exp\{-|\Sigma_2 z - \\
&- \Sigma_2 z' + (k-k')(\Sigma_1 a_1 + \Sigma_2 a_2) - \Sigma_1 a_1|t\} \times \\
&\times I_0\left(\varkappa_{\parallel}|z-z' + (k-k')a - a_1|\sqrt{t^2-1}\right) \times \\
&\times \frac{\Sigma_{a2}}{2} \psi(z' + k'a + a_1),
\end{aligned} \tag{5a}$$

$$\psi(z+k\alpha+\alpha_1)=\sum_{k'=-\infty}^{\infty}\int_0^{\alpha_2}dz'\int_1^{\infty}\frac{dt}{t}\times$$
$$\times\exp\{-|\Sigma_2 z-\Sigma_1 z'+(k-k')(\Sigma_1\alpha_1+\Sigma_2\alpha_2)+$$
$$+\Sigma_1\alpha_1|t\}\,I_0(x_{11}|z-z'+(k-k')\alpha+$$
$$+\alpha_1|\sqrt{t^2-1})\cdot\frac{\Sigma_{s1}}{2}\,\psi(z'+k'\alpha)+$$
$$+\sum_{k'=-\infty}^{\infty}\int_0^{\alpha_2}dz'\int_1^{\infty}\frac{dt}{t}\exp\{-|\Sigma_2(z-z')+$$
$$+(k-k')(\Sigma_1\alpha_1+\Sigma_2\alpha_2)|t\}\times$$
$$\times I_0\left(x_{11}|z-z'+(k-k')\alpha|\sqrt{t^2-1}\right)\times$$
$$\times\frac{\Sigma_{s2}}{2}\,\psi(z'+k'\alpha+\alpha_1). \qquad (5b)$$

(In these equations we again use relative coordinates z and z').

Equations (2) and (5) are homogeneous integral equations of the type

$$\lambda\varphi(z)=\int dz'\,K(z,z')\,\sigma(z')\cdot\varphi(z') \qquad (6)$$

with a symmetric kernel $K(z, z') = K(z', z)$.

The corresponding variational problem which leads to this equation, as is well known, is

$$\lambda=\frac{\int dz\,\varphi(z)\,\sigma(z)\int dz'\,K(z,z')\,\sigma(z')\varphi(z')}{\int dz\,\varphi^2(z)\,\sigma(z)}=\text{extremum} \qquad (7)$$

Let us use the variational method to solve Equations (2) and (5), since it is most suitable for the solution of the problem of homogenizing heterogeneous mediums. This formulation of the variational problem is the most appropriate one, since the limits of integration in (6) and (7) are at infinity, and this causes the functionals in (7) to increase without bound.

It should also be noted that Expression (7) for an infinite homogeneous medium characterized by the constants Σ_s, Σ_a, $\Sigma = \Sigma_s + \Sigma_a$ and κ with a plane neutron source (z = const) at $z = \infty$, can be written

$$\lambda=\frac{\Sigma_s}{2\kappa}\ln\frac{\Sigma+\kappa}{\Sigma-\kappa}. \qquad (8)$$

Determination of κ_g

Let us consider case 1 of the two cases we have previously mentioned. It is simple to set up Expression (7) for Equation (2); we shall vary the function ψ_0, choosing it so that it corresponds approximately to the solution of well-known neutron diffusion problems. Let us set

$$\left.\begin{aligned}\psi_0(z+k\alpha)&=e^{\kappa\perp(z+k\alpha)}\sum_{n=-\infty}^{\infty}C_n\cdot e^{i\frac{2\pi n z}{\alpha}},\\ \psi_0(z+k\alpha+\alpha_1)&=\\ &=e^{\kappa\perp(z+k\alpha+\alpha_1)}\sum_{n=-\infty}^{\infty}C_n e^{i\frac{2\pi n(z+\alpha_1)}{\alpha}},\end{aligned}\right\} \qquad (9)$$

The function $e^{\kappa_\perp z}$ describes the general variation of the neutron flux and corresponds to its asymptotic behavior in the homogenized medium with a neutron source in the $z = \infty$ plane. Onto this function there is superposed a "fine" structure given by the Fourier series. This corresponds to the heterogeneous character of the medium given by the layers of media 1 and 2. In (9) we vary the constants $C_n = C^*_{-n}$ for $n > 0$, maintaining C_0 as the normalizing (arbitrary) amplitude.

If Equation (9) is inserted into (7), we obtain an extremely complex expression for λ. This can be simplified by assuming that $\kappa_\perp \ll \alpha^{-1}$. This assumption, however, is at the same time the condition that makes it possible to homogenize a heterogeneous medium. Only if $\alpha \cdot \kappa_\perp \ll 1$ is it possible to separate the general variation $e^{\kappa_\perp z}$ of the neutron flux from its fine structure given by the periodic modulation of the Fourier series.

Equation (9) can be written

$$\left.\begin{aligned} \psi_0(z+ka) &= e^{\kappa_\perp ka} \cdot \sum_{n=-\infty}^{\infty} C_n e^{i\frac{2\pi n z}{a}}, \\ \psi_0(z+ka+a_1) &= e^{\kappa_\perp ka} \sum_{n=-\infty}^{\infty} C_n e^{i\frac{2\pi n (z+a_1)}{a}}, \end{aligned}\right\} \tag{10}$$

which corresponds even better to the problem of homogenizing a heterogeneous medium.

However even this form of the function ψ_0 is not final. If we were to insert the function of the form given by (10) into (7) according to Equation (2), the quantity λ would have to have the following properties:

1. It would have to be invariant under replacement of $\kappa_\perp$ by $-\kappa_\perp$

2. Translation of the coordinate system in the positive z direction by an amount a_1 should correspond identically to interchange of media 1 and 2 in the expression for λ, as can be seen from the figure. Only such an expression for λ will have physical meaning and determine $\kappa_\perp$ and the Fourier coefficients C_n in an uncontradictory and unique way.

Expression (10) for ψ_0 does not give an expression for λ that has these properties; the necessary function is

$$\left.\begin{aligned} \psi_0(z+ka) &= \operatorname{ch} \kappa_\perp ka \sum_{n=-\infty}^{\infty} C_n e^{i\frac{2\pi n z}{a}}, \\ \psi_0(z+ka+a_1) &\operatorname{ch} \kappa_\perp ka \sum_{n=-\infty}^{\infty} C_n e^{i\frac{2\pi n (z+a_1)}{a}}. \end{aligned}\right\} \tag{11}$$

This function corresponds to the asymptotic neutron flux for equal intensity plane sources at $z = -\infty$ and $z = +\infty$. After some calculation the expression for λ can be written in the form of a fraction $\lambda = u/v$ where u and v are given by the following expressions

$$\left.\begin{aligned} u &= \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} C_p C_q \cdot a_{p,q}, \\ v &= \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} C_p C_q \cdot b_{p,q}, \end{aligned}\right\} \tag{12a}$$

$$a_{p,q} = -\frac{\Sigma_{11}}{4}\left\{\sum_{k=1}^{\infty} 2\,\mathrm{ch}\,(kx_\perp a)\cdot\left[-\left(1+e^{i\frac{2\pi a_1}{a}(p+q)}\right)\times\right.\right.$$

$$\times E_{11}(k, k; p, q) + e^{i\frac{2\pi q a_1}{a}} E_{11}(k-1, k; p, q) +$$

$$+ e^{i\frac{2\pi p a_1}{a}} E_{11}(k+1, k; p, q)] -$$

$$-\left(1+e^{i\frac{2\pi a_1}{a}(p+q)}\right) E_{11}(0, 0; p, q) +$$

$$+ 2e^{i\frac{2\pi p a_1}{a}} E_{11}(1, 0; p, q) +$$

$$+ \frac{a^2}{2\pi^2 iq\,(p+q)}\left(e^{i\frac{2\pi a_1}{a}(p+q)} - 1\right)\times$$

$$\left.\left.\times\left(\frac{\pi}{2} - \mathrm{arctg}\,\frac{\Sigma_1 a}{2\pi q}\right)\right\} + \frac{\Sigma_{12}^2}{4}\cdot e^{i\frac{2\pi a_1}{a}(p+q)}\times\right.$$

$$\times\left\{\sum_{k=1}^{\infty} 2\,\mathrm{ch}\,(kx_\perp a)\left[-\left(1+e^{i\frac{2\pi a_1}{a}(p+q)}\right)\times\right.\right.$$

$$\times E_{22}(k, k; p, q) + e^{i\frac{2\pi q a_2}{a}} E_{22}(k, k-1; p, q) +$$

$$+ e^{i\frac{2\pi p a_2}{a}}\cdot E_{22}(k, k+1; p, q)] -$$

$$-\left(1+e^{i\frac{2\pi a_2 (p+q)}{a}}\right) E_{22}(0, 0; p, q) +$$

$$+ 2e^{i\frac{2\pi p a_2}{a}} E_{22}(0, 1; p, q) + \frac{a^2}{2\pi^2 iq\,(p+q)}\times$$

$$\left.\left.\times\left(e^{i\frac{2\pi a_2}{a}(p+q)} - 1\right)\left(\frac{\pi}{2} - \mathrm{arctg}\,\frac{\Sigma_2 a_2}{2\pi q}\right)\right\} +\right.$$

$$+ \frac{\Sigma_{11}\Sigma_{12}}{2}\left\{\sum_{k=0}^{\infty}\mathrm{ch}\,(kx_\perp a)\times\right.$$

$$\times[-e^{i\frac{2\pi p a_1}{a}} E_{21}(k+1, k; p, q) -$$

$$- e^{i\frac{2\pi q a_1}{a}} E_{21}(k, k+1; p, q) +$$

$$+ E_{21}(k+1, k+1; p, q) +$$

$$+ e^{i\frac{2\pi a_1}{a}(p+q)} E_{21}(k, k; p, q) -$$

$$- e^{i\frac{2\pi p a_1}{a}} E_{12}(k, k+1; q, p) + E_{12}(k, k; q, p) -$$

$$- e^{i\frac{2\pi q a_1}{a}} E_{12}(k+1, k; q, p) +$$

$$\left.+ e^{i\frac{2\pi a_1}{a}(p+q)} E_{12}(k+1, k+1; q, p)\right\}\,, \tag{12b}$$

$$b_{p,q} = \frac{\Sigma_{21}}{2}\cdot\frac{a}{2\pi i\,(p+q)}\left(e^{i\frac{2\pi a_1}{a}(p+q)} - 1\right) +$$

$$+ \frac{\Sigma_{22}}{2}\cdot\frac{a}{2\pi i\,(p+q)}\, e^{i\frac{2\pi a_1}{a}(p+q)}\left(e^{i\frac{2\pi a_2}{a}(p+q)} - 1\right). \tag{12c}$$

The functions $E_{hj}(k, l; p, q)$ are given by the integral

$$F_{hj}(k, l;\ p, q) = \int_0^\infty \frac{dt}{t} \cdot \frac{e^{-\lambda z_1 \varepsilon_1 t}}{\left(\Sigma_h t - i\frac{2\pi p}{a}\right)} \cdot \frac{e^{-\lambda z_2 \varepsilon_2 t}}{\left(\Sigma_j t + i\frac{2\pi q}{a}\right)} \cdot \tag{13}$$

Instead of the complex variation parameters C_n, let us now introduce the real parameters given by

$$C_n = Z_{2n} + iZ_{2n-1},\ C_{-n} = Z_{2n} - iZ_{2n-1}$$

with

$$n > 0 \quad \text{and} \quad C_0 = Z_0. \tag{14}$$

so that $\underline{u}$ and $\underline{v}$ are quadratic forms in the real amplitudes Z_n $(n = 0, \ldots, \infty)$, namely

$$u = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} c_{p,q} Z_p Z_q,\ v = \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} d_{p,q} Z_p Z_q. \tag{15}$$

The coefficients $c_{p,q} = c_{q,p}$ are related to the previous coefficients $a_{p,q}$ by the expressions

$$\begin{aligned}
c_{2n,2k} &= \tfrac{1}{2}[a_{n,k} + a_{k,n} + a_{-n,k} + a_{k,-n}] + \\
&\quad + c_n c_k \\
c_{2n,2k-1} &= i[a_{n,k} + a_{k,n} - a_{-n,k} - a_{k,-n}] + \\
&\quad + c_n c_k \qquad c_{2n-1,2k-1} = \\
&= \tfrac{1}{2}[-a_{n,k} - a_{k,n} + a_{-n,k} + a_{k,-n}] + c_n c_k \\
c_{2n,0} &= (a_{n,0} + a_{0,n}) + c_n c_0 \\
c_{2n-1,0} &= i(a_{n,0} + a_{0,n}) + c_n c_0 \qquad , c_{0,0} = a_{0,0}.
\end{aligned} \tag{16}$$

The same relations hold between the $d_{p,q} = d_{q,p}$ and the $b_{p,q}$.

The requirement that $\lambda = \frac{u}{v}$ have an extremum for variations of the Z_n $(n = 1, \ldots, \infty)$ gives the equations

$$\sum_{p=0}^{\infty} (c_{n,p} - d_{n,p}) Z_p = 0, \quad n > 0, \tag{17}$$

if $\kappa_\perp$ is chosen so that $\lambda = 1$, i.e., so that $u = v$. From this choice of λ as well as from (17) it follows that they are real also for $n = 0$, i.e., in general for $n \geq 0$. The condition that such a system of equations in the Z_n be soluble is the requirement that the determinant of the system vanish, namely

$$D(c_{n,p} - d_{n,p}) = 0. \tag{18}$$

Since the $c_{n,p}$ and $d_{n,p}$ depend on $\kappa_\perp$, this equation can be satisfied if $\kappa_\perp$ is properly chosen. The lowest positive value of the roots of Equation (18) will be the desired reciprocal of the diffusion length for the homogenized heterogeneous medium.

The amplitudes Z_q ($q = 1, \ldots, \infty$) are determined from the equations

$$Z_q = \frac{D_{0q}(c_{n,p} - d_{n,p})}{D_{00}(c_{n,p} - d_{n,p})} Z_0, \quad q = 1, \ldots, \infty, \tag{19}$$

where the D_{0q} and D_{00} are the appropriate minors of the elements in the determinant.

Thus we were able to solve the problem of homogenization of a heterogeneous medium. Equation (18) determines the reciprocal of the diffusion length $\kappa_\perp$; Expression (19) can be used to determine the variation of the neutron flux in media 1 and 2. These values may be used in constructing the asymptotic neutron flux corresponding to a different position of the plane source of monoenergetic neutrons and a different over-all geometry of the heterogeneous medium.

Some questions arise, however, with respect to the above formal solution of the problem of homogenization. For instance, it would be interesting to discover whether the Fourier coefficients given by Equation (19) indicate the discontinuity in the neutron flux at the separation boundary between media 1 and 2, as follows from the results of Bat and Zaretsky [1] for the asymptotic neutron flux. The solution of this problem in general is very difficult due to complexity of the results obtained. Further, it is necessary to prove the existence of the above solution, in particular the existence of $\kappa_\perp$. We shall attempt to do this by solving a simplified homogenization problem.

This time let us choose the variation function ϕ_0 to be

$$\phi_0(z + k\alpha) = e^{z\perp^{k\alpha}}, \quad \phi_0(z + k\alpha + \alpha_1) = Ce^{z\perp^{k\alpha}}. \tag{20}$$

In these expressions the Fourier series $\sum_{n=-\infty}^{\infty} C_n \cdot e^{i\frac{2\pi nz}{\alpha}}$ modulating the neutron flux is replaced by its average value. In medium 1 the mean value is taken as unity, since its absolute value is proportional to the neutron source intensity, which is arbitrary. In medium 2 the mean value is given by the constant C. This constant is also a variational parameter; it is chosen so that λ is an extreme value. The thermal utilization factor of the heterogeneous medium relative to that of medium 1 is

$$f_\perp = \frac{\Sigma_{a1}\alpha_1}{\Sigma_{a1}\alpha_1 + C\Sigma_{a2}\alpha_2} \cdot f_\perp = \frac{C\Sigma_{a2}\alpha_2}{\Sigma_{a1}\alpha_1 + C\Sigma_{a2}\alpha_2}. \tag{21}$$

This expression gives additional physical meaning to the parameter C.

The expression for λ with the Function (20) is easy to obtain from Equations (2) and (7). We also make use of (12), writing λ in the form of the fraction $\lambda = \frac{u}{v}$, where

$$u = \left(\frac{\Sigma_{s1}}{\Sigma_s}\right)^2 \cdot \frac{C^2}{2} \left\{ \sum_{k=1}^{\infty} \operatorname{ch}(k\kappa_\perp \alpha) \times \right.$$
$$\times [-2E_3(k(\Sigma_1\alpha_1 + \Sigma_2\alpha_2)) +$$
$$+ E_3(k(\Sigma_1\alpha_1 + \Sigma_2\alpha_2) - \Sigma_2\alpha_2) +$$
$$+ E_3(k(\Sigma_1\alpha_1 + \Sigma_2\alpha_2) + \Sigma_2\alpha_2)] -$$
$$\left. - \frac{1}{2} + E_3(\Sigma_2\alpha_2) + \Sigma_2\alpha_2 \right\} + \frac{\Sigma_{s1}\Sigma_{s2}}{\Sigma_1\Sigma_2} C \times$$

$$\times\left\{\sum_{k=0}^{\infty}\operatorname{ch}(k\varkappa_{\perp}a)\left[-E_3(k(\Sigma_1a_1+\Sigma_2a_2)+\right.\right.$$
$$+\Sigma_1a_1)-E_3(k(\Sigma_1a_1+\Sigma_2a_2)+\Sigma_2a_2)+$$
$$+E_3(k(\Sigma_1a_1+\Sigma_2a_2))+$$
$$\left.\left.+E_3((k+1)(\Sigma_1a_1+\Sigma_2a_2))\right\}+\right.$$
$$+\left(\frac{\Sigma_{21}}{\Sigma_{11}}\right)^2\cdot\frac{1}{2}\left\{\sum_{k=1}^{\infty}\operatorname{ch}(k\varkappa_{\perp}a)\times\right.$$
$$\times[-2E_3(k(\Sigma_1a_1+\Sigma_2a_2))+$$
$$+E_3(k(\Sigma_1a_1+\Sigma_2a_2)-\Sigma_1a_1)+$$
$$+E_3(k(\Sigma_1a_1+\Sigma_2a_2)+\Sigma_1a_1)]-$$
$$\left.-\frac{1}{2}+E_3(\Sigma_1a_1)+\Sigma_1a_1\right\}, \tag{22a}$$

$$v=\frac{1}{2}\Sigma_{11}a_1+\frac{1}{2}\Sigma_{12}a_2C^2. \tag{22b}$$

If $\varkappa_{\perp}$ is chosen so that $\lambda = 1$, i.e., so that $u = v$, the requirement that λ have an extremum is identical with the requirement that the expression

$$u-v\equiv aC^2+bC+c, \tag{23}$$

have an extremum, where the coefficients a, b, and c, depend on $\varkappa_{\perp}$ and are determined by (22). Expression (23), as is known, will be extremal when

$$C=-\frac{b}{2a}, \tag{24}$$

so that in view of the choice $\lambda = 1$, the value of $\varkappa_{\perp}$ is determined by the equation

$$4ac-b^2=0. \tag{25}$$

Let us go on with an evaluation of these results. Let us assume that α_1 and α_2 are small, so that $u - v$ can be written on the basis of Equations (22)

$$u-v=\frac{1}{2}(\Sigma_{11}a_1)^2[F(\varkappa_{\perp})+A_2]C^2+$$
$$+\Sigma_{11}a_1\Sigma_{12}a_2[F(\varkappa_{\perp})+B]C+$$
$$+\frac{1}{2}(\Sigma_{12}a_2)^2[F(\varkappa_{\perp})+A_1], \tag{26}$$

with sufficient accuracy, where we have introduced the notation

$$F(\varkappa_{\perp})=\sum_{k=1}^{\infty}E_1(k(\Sigma_1a_1+\Sigma_2a_2))\operatorname{ch}(k\varkappa_{\perp}a), \tag{27a}$$

$$A_i=\frac{1}{(\Sigma_{i1}a_i)^2}\left[-\frac{1}{2}-E_3(\Sigma_1a_1)\right]-\frac{\Sigma_{2i}}{\Sigma_{1i}}\cdot\frac{1}{\Sigma_{1i}a_i}, \tag{27b}$$

$$\dot{B} = \frac{1}{\Sigma_1 a_1 \Sigma_2 a_2}\left[\frac{1}{2} + E_3(\Sigma_1 a_1 + \Sigma_2 a_2) - \right.$$
$$\left. - E_3(\Sigma_1 a_1) - E_3(\Sigma_2 a_2)\right]. \qquad (27c)$$

Equation (26) is sufficiently accurate, and becomes more so as a_1 and a_2 become smaller. Therefore, we must also express A_1 and B in the form of Taylor's series, restricting ourselves to the first largest terms (Euler's constant $\gamma = 0.577$):

$$A_1 = -\frac{1}{\Sigma_{s1} a_1} + \frac{3}{4} - \frac{1}{2}\gamma - \frac{1}{2}\ln(\Sigma_1 a_1), \qquad (28a)$$

$$B = \frac{3}{2} - \gamma - \ln(\Sigma_1 a_1 + \Sigma_2 a_2) - \frac{1}{2}\frac{\Sigma_1 a_1}{\Sigma_2 a_2}\ln \times$$
$$\times\left(1 + \frac{\Sigma_2 a_2}{\Sigma_1 a_1}\right) - \frac{1}{2}\frac{\Sigma_2 a_2}{\Sigma_1 a_1}\ln\left(1 + \frac{\Sigma_1 a_1}{\Sigma_2 a_2}\right). \qquad (28b)$$

Then (25) for $\kappa_\perp$ becomes

$$F(\kappa_\perp) = \frac{A_1 A_2 - B^2}{2B - (A_1 + A_2)}, \qquad (29a)$$

and C is given by the formula

$$C = \frac{\Sigma_{s1} a_1}{\Sigma_{s2} a_2} \cdot \frac{B - A_1}{B - A_2} \qquad (29b)$$

If we now allow the thicknesses a_1 and a_2 to approach zero, the sum (27a) can be replaced by an integral, and we obtain

$$F(\kappa_\perp) = \frac{1}{a}\int_0^\infty \mathrm{ch}(s\kappa_\perp) E_1\left(\frac{\Sigma_1 a_1 + \Sigma_2 a_2}{a} \cdot s\right) ds =$$
$$= \frac{1}{2\kappa_\perp^2} \ln \frac{\Sigma_1 a_1 + \Sigma_2 a_2 + \kappa_\perp^2}{\Sigma_1 a_1 + \Sigma_2 a_2 - \kappa_\perp^2}, \qquad (30)$$

so that (29a) gives

$$\frac{\Sigma_{s1} a_1 + \Sigma_{s2} a_2}{2\kappa_\perp^2} \ln \frac{\Sigma_1 a_1 + \Sigma_2 a_2 + \kappa_\perp^2}{\Sigma_1 a_1 + \Sigma_2 a_2 - \kappa_\perp^2} = 1, \qquad (31)$$

and (29b) gives

$$C = 1. \qquad (32)$$

Equations (31) and (32) are exact in the limit $a_1 = 0$, $a_2 = 0$.

Equation (31) is of interest from the theoretical point of view, since it is an exact equation of the kinetic theory and gives the reciprocal $\kappa_\perp$ of the diffusion length corresponding to a homogenized mixture of media 1 and 2 with layer thicknesses a_1 and a_2. At the same time, its continuity shows that the homogenized reciprocal of the diffusion length exists also for a more general heterogeneous medium, as well as that this method of homogenization is entirely valid and has definite physical meaning.

Determination of $\kappa_\parallel$

Let us attempt to show briefly how Equation (5) may be used to find the homogenized reciprocal of the diffusion length (when the neutrons from the source are incident parallel to the layers), since our conclusions differ somewhat from those obtained by Spinrad [3].

As the variation function in Expression (7) for Equation (5) let us start immediately with a simplified one similar to (20):

$$\phi(z+k\alpha)=1,\quad \phi(z+k\alpha+\alpha_1)=C,$$

where the mean value C of the flux is the variation parameter. The corresponding expression for λ may again be written in the form of a fraction $\lambda=\frac{u}{v}$, where

$$u=\left\{\frac{\Sigma_{s1}^2}{4}a_0+\frac{\Sigma_{s1}^2}{2}\sum_{k=1}^{\infty}a_k\right\}C^2+\Sigma_{s1}\Sigma_{s2}\sum_{k=0}^{\infty}b_kC+ \tag{33a}$$

$$+\left\{\frac{\Sigma_{s1}^2}{4}c_0+\frac{\Sigma_{s1}^2}{2}\sum_{k=1}^{\infty}c_k\right\},$$

$$v=\frac{\Sigma_{s1}}{2}\alpha_1+\frac{\Sigma_{s2}}{2}\alpha_2C^2. \tag{33b}$$

The coefficients a_k, b_k, and c_k are in this case given by

$$a_k=\int_0^{\alpha_2}dz\int_0^{\alpha_2}dz'\int_1^{\infty}\frac{dt}{t}\times$$

$$\times\exp[-|\Sigma_2(z-z')+k(\Sigma_1\alpha_1+\Sigma_2\alpha_2)|t]\times \tag{34a}$$

$$\times I_0(\varkappa_{\parallel}|z-z'+k\alpha|\sqrt{t^2-1}),$$

$$b_k=\int_0^{\alpha_1}dz\int_0^{\alpha_2}dz'\int_1^{\infty}\frac{dt}{t}\times$$

$$\times\exp[-|\Sigma_1 z+\Sigma_2 z'+k(\Sigma_1\alpha_1+\Sigma_2\alpha_2)|t]\times \tag{34b}$$

$$\times I_0(\varkappa_{\parallel}|z+z'+k\alpha|\sqrt{t^2-1}),$$

$$c_k=\int_0^{\alpha_1}dz\int_0^{\alpha_1}dz'\int_1^{\infty}\frac{dt}{t}\times$$

$$\times\exp[-|\Sigma_1(z-z')+k(\Sigma_1\alpha_1+\Sigma_2\alpha_2)|t]\times \tag{34c}$$

$$\times I_0(\varkappa_{\parallel}|z-z'+k\alpha|\sqrt{t^2-1}).$$

If $\varkappa_{\parallel}$ is again chosen so that $\lambda=1$, i.e., so that $u=v$, the extreme value of λ for variations of C will be given by the extremum of the expression $u-v$ for variation of C, which is easy to show. The value of C corresponding to this extremum is determined by (24); in Equation (23), of course, $\underline{u}$ and $\underline{v}$ must be replaced by (22). Then $\varkappa_{\parallel}$ will be determined in exactly the same way.

Comparison of (22) and (33) shows that $\varkappa_{\perp}$ and $\varkappa_{\parallel}$ and the corresponding constants C will in general differ for the same heterogeneous medium shown in the figure. Obviously the thermal utilization factors will also differ for this medium, which is indicated in (21). It is interesting to see whether this anisotropy is maintained in the limit $\alpha_1=0$, $\alpha_2=0$.

Therefore, let us now consider our problem for the case when the thicknesses α_1 and α_2 approach zero. If α_1 and α_2 are very small, the sums in Expression (33a) for $\underline{u}$ may be replaced by an integral. We shall show

this in more detail for the sum $\sum_{k=1}^{\infty} c_k$, whose terms are given by Equation (34c). Since $|z - z^0| \leq \alpha_3$ on integrating over $\underline{z}$ and $\underline{z}^0$, we may consider the integrands of (34c) constant. Therefore

$$c_k = \alpha_1^2 \int_1^{\infty} \frac{dt}{t} \exp[-k(\Sigma_1\alpha_1 + \Sigma_2\alpha_2)t] \times \times I_0(x_{\|} k\alpha \sqrt{t^2-1}), \quad (k>0). \tag{35}$$

Let us replace the sum $\sum_{k=1}^{\infty} c_k$ by an integral, writing

$$\sum_{k=1}^{\infty} c_k = \frac{\alpha_1^2}{\alpha} \int_0^{\infty} ds \int_1^{\infty} \frac{dt}{t} \times \times \exp\left[-\frac{\Sigma_1\alpha_1 + \Sigma_2\alpha_2}{\alpha} st\right] I_0(x_{\|} s\sqrt{t^2-1}).$$

Consequently, this integration can be performed in the following way [2]:

$$\sum_{k=1}^{\infty} c_k = \frac{\alpha_1}{\alpha} \int_1^{\infty} \frac{dt}{t} \times \times \frac{1}{\sqrt{\left(\frac{\Sigma_1\alpha_1 + \Sigma_2\alpha_2}{\alpha}\right)^2 t^2 - x_{\|}^2 (t^2-1)}} = \alpha_1 F(x_{\|}). \tag{36}$$

The function $F(\kappa_{\|})$ is the same as that given by (30),

$$F(x_{\|}) = \frac{1}{2x_{\|}\alpha} \ln \frac{\Sigma_1\alpha_1 + \Sigma_2\alpha_2 + x_{\|}\alpha}{\Sigma_1\alpha_1 + \Sigma_2\alpha_2 - x_{\|}\alpha}. \tag{37}$$

Here, of course, it is necessary to assume that $\Sigma_1\alpha_1 + \Sigma_2\alpha_2 > \kappa_{\|}\alpha$.

The remaining sums in Expression (33a) may be represented similarly. Here, however, we shall present only the results:

$$\sum_{k=1}^{\infty} b_k = \alpha_1\alpha_2 F(x_{\|}), \quad \sum_{k=1}^{\infty} a_k = \alpha_2^2 F(x_{\|}). \tag{38}$$

After such transformations, (25) for $\kappa_{\|}$ can be written

$$x(x_{\|}) = -\frac{\left(\frac{1}{\varepsilon_{s1}\alpha_1} - \frac{c_0}{2\alpha_{\|}}\right)\left(\frac{1}{\Sigma_{s2}\alpha_2} - \frac{a_0}{2\alpha_{\|}}\right) - \left(\frac{b_0}{\alpha_1\alpha_2}\right)^2}{\frac{1}{\Sigma_{s1}\alpha_1} + \frac{1}{\Sigma_{s2}\alpha_2} - \frac{c_0}{2\alpha_{\|}} - \frac{a_0}{2\alpha_{\|}} + \frac{2b_0}{\alpha_1\alpha_2}} \tag{39}$$

On the right side of this equation it is necessary to go the limit $\alpha_1 = 0$, $\alpha_2 = 0$. In view of the fact that

$e^{\kappa|z|t} > I_0(\kappa|z|\sqrt{t^2-1})$ for $\kappa > 0$, $t \geq 1$, for sufficiently small values of α_1 and α_2 Equation (34) gives

$$\left.\begin{aligned} a_0 &< a_1^2\left[\frac{3}{2} - \gamma - \ln(\Sigma_1 - \varkappa_{\parallel})a_1\right], \\ c_0 &< a_2^2\left[\frac{3}{2} - \gamma - \ln(\Sigma_2 - \varkappa_{\parallel})a_2\right], \\ b_0 &< a_1 a_2\left[\frac{3}{2} - \gamma - \ln(\Sigma_1 a_1 + \Sigma_2 a_2 - \varkappa_{\perp} a)\right] - \\ &- \frac{1}{2}\frac{\Sigma_1 - \varkappa_{\parallel}}{\Sigma_2 - \varkappa_{\parallel}} a_1^2 \ln\left(1 + \frac{\Sigma_2 - \varkappa_{\parallel}}{\Sigma_1 - \varkappa_{\parallel}} \cdot \frac{a_2}{a_1}\right) - \\ &- \frac{1}{2}\frac{\Sigma_2 - \varkappa_{\parallel}}{\Sigma_1 - \varkappa_{\parallel}} a_2^2 \ln\left(1 + \frac{\Sigma_1 - \varkappa_{\parallel}}{\Sigma_2 - \varkappa_{\parallel}} \cdot \frac{a_1}{a_2}\right). \end{aligned}\right\} \quad (40)$$

if, of course, $\Sigma_i > \kappa_{\parallel}$ for $i = 1, 2$. This means that in the limit $\alpha_1 = 0$, $\alpha_2 = 0$ Expressions (37), (39), and (40), lead to the relation

$$\frac{\Sigma_{11}a_1 + \Sigma_{12}a_2}{2\varkappa_{\perp} a} \cdot \ln \frac{\Sigma_1 a_1 + \Sigma_2 a_2 + \varkappa_{\perp} a}{\Sigma_1 a_1 + \Sigma_2 a_2 - \varkappa_{\perp} a} = 1. \quad (41)$$

In a similar way it can be shown that the corresponding value of the mean flux is

$$C = 1. \quad (42)$$

The result obtained is in contradiction with that of Spinrad [3] since we have shown that the homogenized reciprocals $\kappa_{\perp}$ and $\kappa_{\parallel}$ of the diffusion lengths are equal for the heterogeneous medium shown in the figure in the limit $\alpha_1 = 0$, $\alpha_2 = 0$. Both values of κ satisfy the same equations of kinetic theory (31) and (41). ($C = 1$ for both cases also). As is well known, however, Equations (31) and (41) have a single, real (positive) root, so that $\kappa_{\perp} = \kappa_{\parallel}$. This root satisfies the requirement $\Sigma_1 \alpha_1 + \Sigma_2 \alpha_2 > \kappa \alpha$.

SUMMARY

1. The constants of the homogenized heterogeneous medium depend in general on the mutual orientation of the neutron current density and the anisotropy of the heterogeneous medium.

2. The constants of the homogenized heterogeneous medium do not depend on the mutual orientation of the neutron current density and the anisotropy of the heterogeneous medium when the thickness of the layers which go to make up this medium approach zero. These constants then become equal to those of the corresponding homogeneous mixture.

This second conclusion is in contradiction to that of Spinrad [3]. In view of the fact that we arrived at this conclusion by the kinetic theory, and not on the basis of elementary neutron diffusion theory as did Spinrad, we consider our results correct.

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CROSS SECTION FOR THERMAL-NEUTRON ABSORPTION AND RESONANCE ABSORPTION INTEGRAL IN Pu^{240}

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Using the oscillating reactor method the cross section for the absorption of thermal neutrons neutrons and the resonance absorption integral in the isotope Pu^{240} have been measured. The measurements were performed at two positions inside the heavy-water reactor of the Academy of Sciences, USSR: in the lattice and in the thermal column. Plutonium samples with three different abundances of the isotopes Pu^{239}, Pu^{240} and Pu^{241} were used. The oscillations in the power level produced by excursions of the samples were recorded with a recording potentiometer. To avoid self-shielding effects, the dependence of sample effect on plutonium weight was investigated. To determine the spectral composition of the neutron flux in the lattice and in the column the cadmium ratio for indium was measured. The values for the cross sections in Pu^{240} are as follows: in the column – (560 ± 35) barns, in the lattice – (1010 ± 120) barns. The cross section for absorption of thermal neutrons in Pu^{240} was found to be σ^T = (460 ± 45) barns and the resonance absorption integral – Σ = (10,000 ± 2,800) barns. The value obtained for the thermal neutron cross section agrees with a contribution in the thermal cross section from a resonance level at $E_0 \approx 1$ ev. This indicates the absence of strong levels in Pu^{240} in the energy region from 1 ev down to negative values and also indicates that in the thermal-neutron region the cross section in Pu^{240} varies according to a 1/v law.

INTRODUCTION

The properties of the isotopes Pu^{240} and Pu^{241}, which accumulate in extended fuel cycles, are of great importance in reactor calculations. The properties of Pu^{241} are now known with a sufficient degree of accuracy. The behavior of the fission cross section in Pu^{241} has been investigated in detail in the energy region from 0.01-700 ev [1, 2]. The fission cross section in Pu^{241} measured for a thermal spectrum [3, 4] is in agreement. The cross sections for absorption and radiative capture of reactor neutrons by this isotope [3, 4], and the effective number of secondary neutrons per absorption event [3, 5] measured in the early experiments, are found to be in good agreement. The data on the cross sections in Pu^{240} [3, 4, 6], however, exhibit a large spread and uncertainty because of the strong effect of self-shielding, due to the strong resonance in the absorption cross section at an energy of about 1 ev, and the uncertainty in the spectral composition of the neutron flux in which the experiment was performed. The resonance absorption integral in Pu^{240} has been measured only recently [7]. To eliminate the effect of self-shielding in Pu^{240}, it is necessary to measure the effects in small amounts of this isotope. It was decided to make use of the oscillating reactor method [8, 9], which has a high sensitivity as compared with other methods; the method was applied to various "multiplier" elements. The measurements were performed in the heavy water reactor of the Academy of Sciences, USSR, at two points, differing in spectral composition of neutron flux: inside the reactor lattice (a Fermi spectrum plus a thermal spectrum) and in the so-called thermal column of the reactor [10] (almost a thermal spectrum) (Fig. 1).

Method of Measurement

Plutonium with three different abundances of the isotopes Pu^{239}, Pu^{240} and Pu^{241} was available. The samples were inserted into the reactor and withdrawn periodically. The reactor was operated without the automatic control system; hence, the power level oscillated with a period equal to the oscillation period of the samples and with a relative amplitude proportional to the change in the reactivity of the reactor due to the introduction of the samples. The oscillations in the reactor power level produced oscillations in the neutron flux incident on the neutron chamber. The fluctuating chamber current was fed to a dc amplifier and then recorded with an electronic recording-potentiometer.

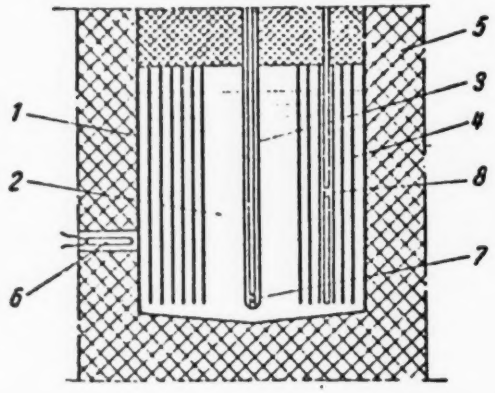

Fig. 1. Diagram of the experiment: 1) Reactor tank; 2) heavy water; 3) vertical experimental tube; 4) uranium channel; 5) reflector (graphite); 6) neutron chamber; 7) position of sample in thermal well; 8) position of sample in reactor lattice.

We shall use the notation $\alpha^{(i)}$ ($i = 0, 1, 2$) to designate the efficiency of the sample of the the i-th isotopic component. By sample efficiency we mean the effect of a given sample on the reactor reactivity, that is, the amplitude of the current fluctuations in the neutron chamber, in conventional units, divided by the weight of plutonium in the sample. In this procedure the efficiency of the samples of one of the isotopic components (index 0) is taken as unity ($\alpha^{(0)} = 1$). It can be shown that the cross section for neutron absorption in the isotope Pu^{240} (σ_{40}) can be expressed in terms of the absorption cross section σ, the effective number of secondary neutrons ν_{eff} and the concentration N of the isotopes Pu^{239} and Pu^{241}:

$$\sigma_{40}^{(i)} = \frac{(\nu_{eff\,39} - \mu)\,\sigma_{39}\,(N_{39}^{(i)} - \alpha^{(i)} N_{39}^{(0)})}{\mu\,(N_{40}^{(i)} - \alpha^{(i)} N_{40}^{(0)})} + \frac{(\nu_{eff\,41} - \mu)\,\sigma_{41}\,(N_{41}^{(i)} - \alpha^{(i)} N_{41}^{(0)})}{\mu\,(N_{40}^{(i)} - \alpha^{(i)} N_{40}^{(0)})}, \quad (1)$$

where μ is a correction factor which takes into account the different effects on the reactor reactivity [11] due to the fast neutrons which are produced and the slow neutrons which are absorbed.

By measuring the efficiency of plutonium samples with three different isotopic concentrations it is possible to determine σ_{40} independently using two combinations of isotopic components ($i = 1$ and 2).

The measurements performed in the thermal column and in the lattice determine the cross sections in Pu^{240} for the specta $\sigma_{40\,well}$ and $\sigma_{40\,lat}$, respectively.

The cross sections which are found are related to the cross section for thermal-neutron absorption σ_{40}^{T} and the resonance absorption integral Σ_{40} by the following:

$$\left.\begin{aligned} \sigma_{40\,well} &= \sigma_{40}^{T} + k_1 \Sigma_{40}, \\ \sigma_{40\,lat} &= \sigma_{40}^{T} + k_2 \Sigma_{40}, \end{aligned}\right\} \quad (2)$$

where k_1 and k_2 are coefficients which take account of the ratio between the fluxes of resonance and thermal neutrons in the column and in the lattice. To determine the coefficients k_1 and k_2 the cadmium ratio for indium was measured in both spectra.

Description of the Experimental Setup

Plutonium samples. The isotopic composition of the plutonium, as determined with a mass-spectrometer by Kukavadze, are shown in Table 1.

TABLE 1
Isotopic Composition of the Plutonium Which was Studied

Index of the isotopic component	Contents of the isotopes by %		
	Pu^{239}	Pu^{240}	Pu^{241} *)
0	98.44±0 03	1.56±0.03	—
1	91.87±0.06	7.13±0.06	0.97±0.02
2	79 5 ±0.2	17.2 ±0.2	3.14±0.05

*) Measured content corrected for Pu^{241} decay.

The plutonium samples were prepared as solutions $Pu(SO_4)_2$ in heavy water. To dissolve the $Pu(SO_4)_2$ the heavy water was acidified by a small amount of sulfuric acid (0.5 N) in which the hydrogen was replaced by deuterium. To eliminate the effect of self-shielding, three samples of each isotopic component were prepared. The weight of plutonium in these was related approximately as 1:2:3. The Pu^{239} content in all samples of the same weight group was chosen to be approximately the same in order to eliminate the effect of neutron shielding, (Table 2). The volume of the solution in each sample was 24 cm^3. The plutonium content in the original solutions was determined by two methods of titrating Pu^{3+} with cerium sulfate: potentiometric titration and titration with orthophenanthroline. This work was performed by Laptev.

The containers were made from a fluoroplastic (teflon) because of its small cross section for neutron absorption and its chemical stability.

Oscillator. The oscillator consisted of a dc motor and pulley on which was wound a length of heavy wire. The sample was suspended from the end of the wire by a fluoroplastic hook. The voltage was periodically applied to the motor through a mechanical time-delay relay, raising and lowering the sample inside a vertical channel in the reactor. The absorption limits of the sample were fixed by a reversing switch, connected to special limit switches which were linked to the motor shaft. The sample was in the reactor for 18 sec and then withdrawn from the reactor for 18 sec. The time required for raising and lowering the sample was the same, 2 sec. Thus, an oscillation cycle lasted for 40 sec. The amplitude of the oscillation was 2.5-3.5 m. The equilibrium position of the sample in the thermal column was displaced downward with respect to the center of the reactor in order to cause changes in the most concentrated thermal-neutron flux (because of the proximity of the reflector) (Fig. 1). In the upper position the sample was far from the core, at the exit of the reactor shield. Hence, the neutron scattering in the sample, observed by Hughes [9] in a sample in the reactor, did not occur.

TABLE 2
Weight of Plutonium in the Samples.

Index of the isotopic component	Weight group					
	I		II		III	
	No. of sample	Wt. mg	No. of sample	Wt. mg	No. of sample	Wt. mg
0	1	17.73	4	35.95	7	53.92
1	2	20.04	5	39.89	8	59.94
2	3	23.03	6	45.85	9	68.72

Neutron chamber. The fluctuations in reactor power were recorded with a BF_3 ionization chamber. To obtain sufficient currents at the small reactor power level used in the present work, the chamber was located in one of the lateral channels intersecting the reactor tank (Fig. 1).

Amplifier and recording instruments. The chamber current was amplified with a precision, balanced, dc amplifier with inverse feedback. The amplified signal was recorded with an electronic recording-potentiometer.

The steady-state power level in the reactor was indicated by a straight line on the potentiometer chart. Oscillations of the reactor power produced proportional deflections of the pen of the potentiometer (Fig. 2).

Measurements

As indicated above, in addition to plutonium, the samples contained heavy water with sulfuric acid; to eliminate effects due to these materials and the container, samples were made up containing a solution of sulfuric acid in heavy water with exactly the same normality as the sample being studied (these were the so-called background samples). The effects in these were subtracted from the effects observed in the plutonium samples.

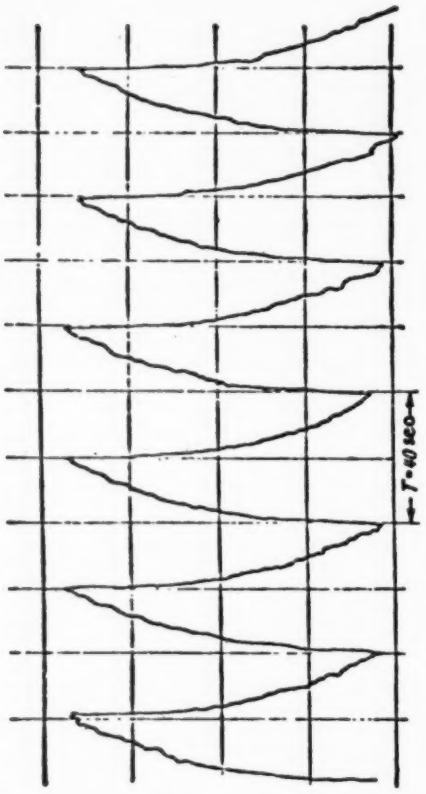

Fig. 2. Chart of the power oscillations produced by oscillations of a sample of III weight group in the thermal column of the reactor.

A set of measurements consisted of successive oscillations of the samples in which each sample oscillated no less than ten times without stop and was then replaced by another sample and so on. The maximum amplitude of the power-level oscillations (for samples of the III weight group) was less than 0.5% of the steady state power level.

The measurements were performed at low reactor power levels (approximately 300 watts) without circulating heavy water through the heat exchanger. Before starting the experiments the reactor was kept in a nonoperative state for 1-2 days. The temperature of the heavy water under these conditions was 33-35° C. Changes in atmospheric pressure had no effect on reactivity since the reactor tank is sealed.

Linearity check. To check the linearity of the entire system (reactor-chamber-amplifier-recorder) samples of boron in the form of a solution of boric acid in heavy water were used. These samples varied widely in weight. In this procedure no departure from linearity was observed (within an accuracy of ± 3%) up to a boron weight of 8 mg, which is equivalent to oscillations corresponding to 160 mg of plutonium. Measurements of the efficiency of the plutonium samples of different groups also verified that the system was linear to within an accuracy of ± 2%.

Dependence of results on oscillation period. Inasmuch as the oscillation in reactor power level is exponential in nature (Fig. 2), it was necessary to verify that the results were independent of oscillation period. To do this a series of measurements were performed in which the oscillation period was one-half the normal period. The results of the measurements were the same within the accuracy of the measurements. However, at a period of 20 sec the oscillation amplitude became rather small, particularly for the I weight group; the amplitude of the observed oscillations was comparable to fluctuations of the current in the input resistance of the amplifier. In this case the errors in the measurements were larger. However, a period much larger than 40 sec yields only an insignificant improvement in the accuracy (because of the exponential nature of the oscillation) and requires excessive time for making measurements.

Determination of the coefficients k_1 and k_2. To determine the coefficients k_1 and k_2 in Equation (2), the cadmium ratios were measured in indium foil. The indium foils were irradiated at the same location as the

plutonium samples. The thickness of the cadmium was 0.25 mm; this implies an effective lower limit on the spectrum of neutrons penetrating the cadmium of 0.4 ev. The activity of the indium foils was measured with a halogen counter for 30-60 min. The cadmium ratio for the indium in the thermal column was 6.5 and in the reactor lattice, 2.0. The coefficients k_1 and k_2 were calculated by Ioffe, using this data, and the following results were obtained:

$$k_1 = 0.010,$$

$$k_2 = 0.055.$$

For these values the thermal Maxwellian spectrum overlaps the Fermi spectrum at an energy of 0.2 ev.

Results of the Measurements

The results of the measurements of the sample efficiency, averaged over several series of measurements, are shown in Fig. 3 and in Tables 3 and 4.

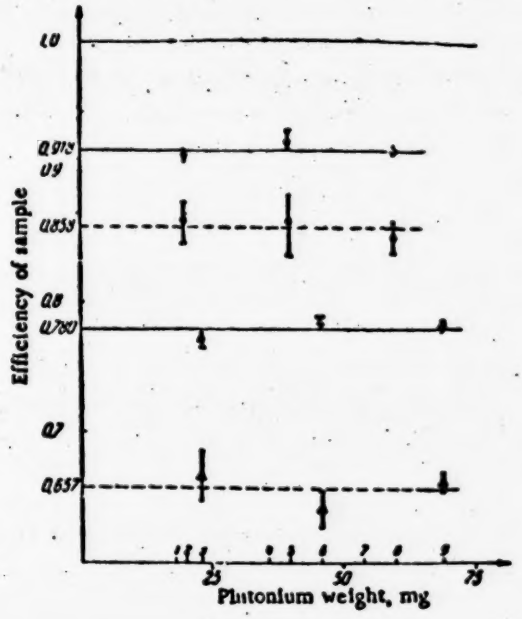

Fig. 3. Efficiency of the samples.
O – Efficiency of the isotopic component with index 1;
Δ – efficiency of the isotopic component with index 2;
— – efficiency in the thermal column;
---— efficiency in the lattice.
The numbers under the abscissa axis designate the number of the sample.

It is apparent from these results that the self-shielding can be neglected in samples of the weight which was chosen, in agreement with the theoretical estimate. If self-shielding did occur, the straight line drawn through the efficiency values of the samples of different weight groups in Fig. 3 would not be parallel to the abscissa axis. Values of the efficiencies, averaged over all weight groups, were substituted in Equation (1). The average values of $\sigma_{4\delta}$ (for $i = 1$ and 2) were found to be:

$$\sigma_{40\,well} = (560 \pm 35) \text{ barns}$$

$$\sigma_{40\,lat} = (1010 \pm 120) \text{ barns}$$

The indicated errors are determined by the spread between the values for i = 1 and i = 2 and may be explained by uncertainties in the determination of the value of $\alpha^{(i)}$, the inaccuracy in the values of the constants for Pu^{241} and the inaccuracy in the measurement of the isotopic composition of the sample and the weight of the sample.

TABLE 3

Efficiency of the Samples in the Thermal Column.

	Weight group			Average value
	I	II	III	
$\alpha^{(0)}$	1	1	1	1
$\alpha^{(1)}$	0.913±0.005	0.923±0.005	0.917±0.003	0.918±0.003
$\alpha^{(2)}$	0.772±0.005	0.785±0.005	0.782±0.003	0.780±0.004

TABLE 4

Efficiency of the Samples in the Lattice.

	Weight group			Average value
	I	II	III	
$\alpha^{(0)}$	1	1	1	1
$\alpha^{(1)}$	0.862±0 015	0.861±0.022	0 850±0.013	0.858±0.010
$\alpha^{(2)}$	0.667±0.019	0.642±0.015	0.662±0.007	0.657±0.008

TABLE 5

Value of the Constants Used in Calculating σ_{40}.

Constants	For the thermal well	For the lattice
σ_{a9}	1140 barns	1340 barns
σ_{a1}	1290 barns	1520 barns
ν_{eff} [10]	2.03 [11]	1,99
ν_{eff}	2.2 [3], [5]	2,2 [3], [5]
β	0.987 [11]	1,281

The values of the constants for Pu^{239}, Pu^{241}, and the correction factor used in calculating the cross section in Pu^{240} are shown in Table 5.

The values of the cross section for Pu^{239} and Pu^{241} in the thermal column were taken as the average for a Maxwellian spectrum with a mean temperature of 305° K; in this procedure the thermal values and the cross sections given in [1, 2, 6] were used and the ratio of the absorption cross section to the fission cross section in Pu^{241} was taken from [3]. The value of the cross sections in the lattice was found from the formula

$$\sigma_{lat} = \sigma^T + 0.055\,\Sigma,$$

where σ^T is the thermal cross section for a Maxwellian spectrum with a mean neutron temperature of 380° K, Σ is the resonance absorption integral with a lower limit of 0.2 ev which was found by integrating the curve showing the dependence of cross section on energy given in [1, 2, 6].

The value of $\nu_{eff\,39}$ for the lattice was calculated by taking into account the reduction in ν_{eff} in Pu^{239} for epithermal neutrons. The value of μ in the lattice was calculated from Equation (2) of [11] using the new values of the constant. Substituting the obtained values of $\sigma_{40\,well}$, $\sigma_{40\,lat}$, k_1 and k_2 in Equation (2), it is possible to determine the following values for the constants being sought:

$$\sigma^T_{40} = (460 \pm 45) \text{ barns}$$
$$\Sigma_{40} = (10\,000 \pm 2800) \text{ barns}$$

The lower limit in the resonance absorption integral is 0.2 ev.

DISCUSSION OF RESULTS

The cross section for the absorption of thermal neutrons in Pu^{240} which was obtained, is consistent with a contribution in the thermal cross section from a resonance level with an energy $E_0 = 1.075$ ev [12]. This agreement is an indication that there are no other strong levels in Pu^{240} in the energy region below E_0 right up to negative energy values. The absence of levels for energies below 1 ev in Pu^{240} is in agreement with the data for the mean distances between levels of fissile heavy isotopes; this distance reaches 10-20 ev. This, in turn, indicates that the cross section in Pu^{240} in the thermal-energy region varies in accordance with a 1/v law. Thus, the obtained value of the cross section in Pu^{240} may be assumed to pertain to a thermal neutron energy $E_T = 0.025$ ev.

The method of measurement of the cross sections which was used made it possible to eliminate neutron self-shielding in the sample and also to separate the thermal cross section in Pu^{240} from the resonance absorption integral. The value of the resonance absorption integral is in good agreement with the value obtained by Erozolimsky et al. [7] by another method and also with an estimate made on the basis of the level parameters for $E_0 = 1.075$ ev [12].

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Note. After this paper was submitted for publication a short report [13] appeared on the measurement of the Pu^{240} constants by a mass-spectrometer method. The data on the resonance integral is in agreement with the data of the present work and [7]; the data on the cross section for thermal neutron absorption, however, does not agree with the data of the present work. It should be noted, however, that the determination of the cross section for thermal neutrons by a mass-spectrometer method has a number of shortcomings.

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MEASUREMENT OF THE EFFECTIVE NUMBER OF SECONDARY NEUTRONS FROM Pu^{239} IN THE INCIDENT NEUTRON ENERGY REGION FROM 7 TO 30 EV

I. V. Kurpichnikov, V. V. Okorokov, and S. I. Sukhoruchkin

Measurements have been made of the energy dependence of the quantity $\nu_{eff} = \nu \frac{\sigma_f}{\sigma_a}$ (where ν is the average number of fast neutrons per fission event, σ_f is the fission cross section and σ_a is the absorption cross section) in Pu^{239} for the energy region ranging from 7 to 30 ev. A knowledge of this quantity is of interest both for practical reactor calculations and in the theory of nuclear fission. The measurements consist of counting the number of fast fission neutrons emitted from a sample of Pu^{239} with slow neutrons incident upon it. A higher resolving power in the neutron collector has made it possible to measure the ratio $\frac{\Gamma_f}{\Gamma_\gamma}$ for the separate resonances. In this connection it is assumed that ν remains constant over the entire measurement region. From the data of the present work it is found that the fission widths of ten levels in Pu^{239} vary strongly from one to another and that they may be grouped.

The quantity $\nu_{eff} = \nu \frac{\sigma_f}{\sigma_a}$ (where ν is the average number of secondary neutrons emitted per fission event, σ_f and σ_a are the cross sections for fission and absorption, respectively) is of great value both in connection with practical reactor calculations as well as in formulating a theory for the fission process in heavy nuclei. The latter situation stems from the fact that a knowledge of the energy dependence of ν_{eff} in conjunction with information on the total widths and the neutron widths (Γ and Γ_n) of the levels makes it possible to estimate the fission widths Γ_f and the radiative widths Γ_γ for the various levels of excited heavy nuclei. The fact that the quantity Γ_γ remains relatively constant for different levels of the same isotope while the quantity Γ_f experiences a marked change has been noted by a number of authors, [1-4] and has received a theoretical explanation in a number of papers [5-7]. The present investigation was undertaken in order to acquire more detailed information on the behavior of the quantities ν_{eff}, Γ_f and Γ_γ in the isotope Pu^{239}.

Experimental Setup

The method of measuring ν_{eff} has been proposed earlier by one of us and described in [4]. The measurements were carried out with the neutron selector described in [8].

In changing over to measurements with a selector channel width of 2 μ sec but with the same arc firing time, it was discovered that when the trigger of the time analyzer is synchronized with the arc voltage in the ion source (as was done in [4-8]) the spread in ion accelerating time in the cyclotron (~ 1.5 μ sec) reduces the resolving power of the selector. To maintain the resolution corresponding to a channel width of 2 μ sec a special scheme was developed; this is a shock-excited, quartz-crystal master-oscillator which provides all the required time triggers in the analyzer (the channel positions, the channel widths, etc.). The intense bursts of γ-rays from the beryllium target, which occur when it is struck by accelerated deuterons, are detected by a special scintillation counter with a plastic scintillator. After being shaped in an integrating circuit, the pulses from the photo-multiplier are used to trigger the master oscillator. Using this scheme the time analyzer is reliably synchronized by the neutron pulse.

In Fig. 1 is shown a diagram of the collimator scheme and the arrangement of the sample and the neutron detector for the measurement of total cross sections with a "flight" distance of 15.6 m. For a major fraction of the total path from the moderator to the neutron detector the beam moves in a tube (collimator) which is evacuated in order to reduce neutron scattering due to air. At the end of the collimator there are slow-neutron detectors – 15 proportional counters, 15 cm long and 3.5 cm in diameter filled with enriched BF_3 to a pressure of 500 mm Hg.

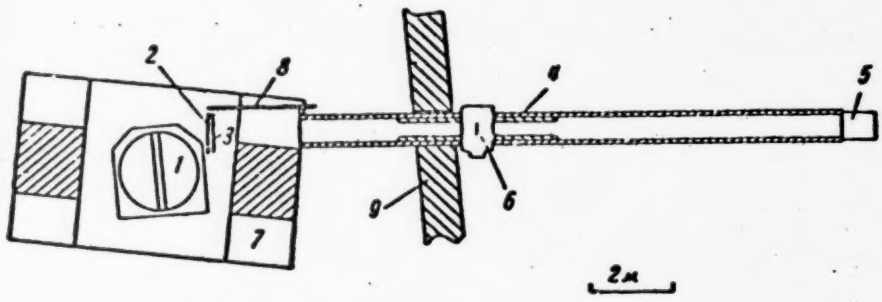

Fig. 1. Diagram of the collimator and layout of the apparatus.
1) Cyclotron chamber; 2) shield of B_4C; 3) paraffin moderator; 4) collimator; 5) detector; 6) sample; 7) cyclotron magnet; 8) supplementary shielding; 9) cyclotron shield.

In making the fission cross section measurements, the detection apparatus shown in Fig. 2 was required. Fast neutrons, produced in fission of nuclei of the sample, are detected by the recoil protons using a scintillation counter. In the present work a sample of Pu^{239} with an area 10 x 20 cm^2 and 0.5 g/cm^2 thick was used. The counter efficiency was enhanced through the use of a photomultiplier with a large photocathode surface and through the use of paraffin as the hydrogenous material.

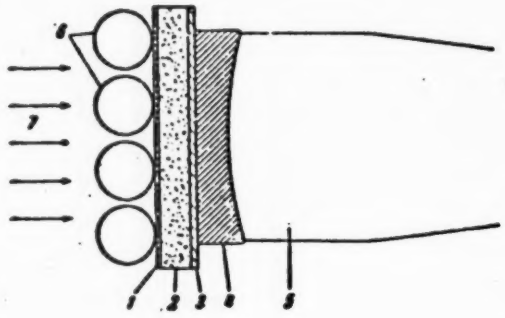

Fig. 2. Detection apparatus.
1) Sample; 2) B_4C; 3) lead; 4) ZnS + paraffin scintillator; 5) photomultiplier; 6) gas counter; 7) neutron beam.

The flux of slow neutrons incident on the sample was measured with four gas counters, 2.5 cm in diameter and 19 cm long, filled with BF_3 (unenriched) to a pressure of 300 mm Hg. These counters, characterized by small self-shielding effects in the thermal region, were placed directly in front of the sample and used to normalize the obtained results with respect to ν_{eff} in the thermal region.

Analysis of the Results of the Measurements

The quantity ν_{eff} was calculated from the formula

$$\nu_{eff} = \frac{N_f \cdot \varepsilon}{N \cdot \sqrt{E}\,(1-T)},$$

where N_f is the recorded number of fission neutrons in a selector channel; $N\sqrt{E}$ is the flux of slow neutrons; $T = e^{-n\sigma_t}$ is the transmission of the sample; n is the number of atoms in 1 cm^2 of the sample; σ_t is the total cross section for the isotopes being studied; ϵ is a quantity which takes into account the efficiency of the counters.

The quantity N_f appears as the difference between the number of counter pulses for fast neutrons recorded with the selector and for the background. This background, produced chiefly by spontaneous fission in Pu^{240}, reaches 30% of the total number of pulses for measurements in the region from 7 to 30 ev and becomes as high as 60% for measurements in the region from 13 to 250 ev. The determination of the magnitude of the background (which is constant for all channels) was carried out in two ways. The first consisted of the measurement of the number of pulses from the fast-neutron counter with the cyclotron shut down, using the last scale during the measurement. In the second method, all pulses, counted by the selector in the region between resonances at energies E_0 = 8 ev and 11 ev, were ascribed to the background. Both methods gave approximately the same results.

Since the measurements of the transmission and the fission effects were carried out with somewhat different resolution, the quantity ν_{eff} for each channel was not calculated, but an average value of ν_{eff} was found for the separate resonances by summing the readings for a number of channels

$$\nu_{eff} = \frac{\epsilon \Sigma N_f}{\Sigma N \sqrt{E}(1-T)}.$$

In addition, the resonances at 10.95 and 11.90 ev were separated graphically as was the group of resonances around 15 ev (14.1, 14.7, 15.5 ev). The values of ν_{eff} determined in this fashion were normalized to the values of ν_{eff} in the thermal region. For this purpose supplementary measurements were carried out; these consisted of the direct comparison of the fission effects for thermal and resonance neutrons. Here, use was made of measurements for a "flight" distance of 15.6 m both for resonance and thermal neutrons (with the use of recycled slow neutrons) and measurements with a flight distance of 1 m for the same moderator (5 cm of paraffin).

Measurements were made of the fission effect using a flight distance of 2 m and a sample thickness of 4.5 g/cm^2. In this case the neutron's energy region from 0.2 to 30 ev was covered again but the recycled neutrons were absorbed by a thin boron filter. The number of resonance neutrons captured in the sample was found by a calculation in which use was made of the quantity $\sigma_0 \Gamma^2$ from [8, 9]. Because of the inadequate resolution of the selector the quantity ν_{eff} can only be given as an average quantity in the energy region from 6.5 to 27 ev. This value is in agreement with the average value of ν_{eff} found by the first normalization method. It was found that in addition to the resonances there is a factor in the cross sections for absorption and fission which changes slowly with neutron energy and which should be ascribed either to an actual remote "negative" level or to some other effect (over-lapping of wide levels or interference between levels). The contribution due to this effect was taken into account in the normalization.

The absorption cross section was determined from the formula

$$\sigma_a = -\frac{1}{n} \ln T - \sigma_s$$

where σ_s = 10 barns is the cross section for the potential scattering and the cross section for fission was determined from the formula $\sigma_f = \frac{N_f \cdot \epsilon_a}{N\sqrt{E}(1-T)}$ (in relative units). The normalization results were used in converting to absolute units.

Results of the Measurements

In Fig. 3 are shown the results of the measurements of $\sigma_a\sqrt{E}$ and $\sigma_f\sqrt{E}$. The quantity ν was taken to be 2.9 for the entire energy region. A number of authors have verified by direct measurement [10-12, 20]

that the quantity ν remains constant in the region from 0.25 to 0.5 ev. Comparing the results of the present work with the results of the measurement of the ratio $\frac{\sigma_f}{\sigma_a}$ using a fission chamber [13], it may be concluded that ν in Pu^{239} resonances does not change to any great extent as compared with the thermal value although the accuracy in the measurement of $\frac{\sigma_f}{\sigma_a}$, as was indicated by the authors of this work, was poor.

Resonance Parameters for Pu^{239} Levels

E_0, ev	$\frac{\nu_{eff}}{\nu_{eff\,T}}$, %	Γ [14] mv	Γ [13] mv	Γ [9] mv	Γ mv	Γ_f mv	Γ_γ mv
0.3	80± 5	98±3	100±5	105 [15]	100±2	56±5	44±4,5
7.80	50± 3	77±13	67±10 44± 7	37±12	42±6	19±3	23±5
10.95	78± 3	146±15	130±40	150±20	130±15	77±10	53±11
11.90	48± 4	52± 8	90±30	85±15	52±5	26±4	26±9
14.30	41± 8	131±22	~70	60±10	60±10	17±5	43±11
14.70	60± 4	80±13	~40	110±30	80±15	40±9	40±7
15.50	112±22	200±70	—	—	200±70	155±80	—
17.65	48± 5	85±11	~150 90±35	80±10	82±8	27±5	55±8
22.30	79± 8	105±22	~60	55± 6	55±5	31±8	24±6
26.10	39±18	91±13	—	140±30	115±17	31±19	84±25

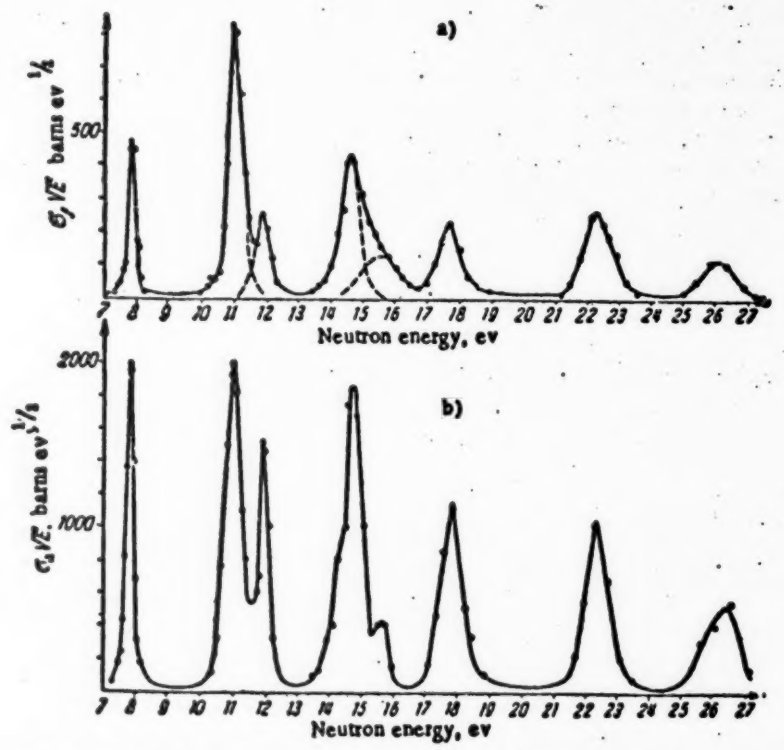

Fig. 3. Results of the measurements of the energy dependence of a) fission cross section; b) absorption cross section.

In the table, in addition to the results of the measurements of ν_{eff} there are shown the total widths for the resonance levels Γ, obtained in [9, 13, 14] and also the quantity Γ which we have used in calculations of the fission widths of the levels Γ_f and the radiative widths Γ_γ. In doing this, we neglect the very small neutron width of the level Γ_n. In the calculations we have used values of the thermal fission cross section and the absorption cross section of 729 and 1032 barns, respectively [19].

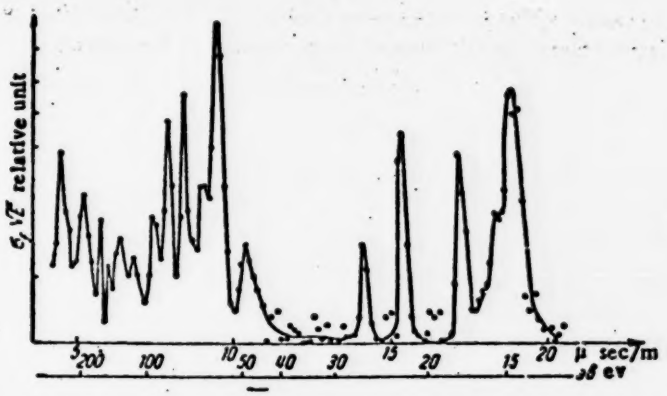

Fig. 4. The function $\sigma_f \sqrt{E}$ in the energy range from 13 to 250 ev without corrections for sample thickness.

The errors indicated in the table take into account only the statistical spread and do not include any possible systematic uncertainty in the normalization, which may reach 15%.

The fission effect was measured separately in the region from 13 to 250 ev. The results are shown in Fig. 4. Comparison with the absorption data [8, 9] indicates that the majority of strong resonances in Pu^{239} have a sizeable fission width. The exceptions are the resonances at energies of 42 and 44 ev (which possibly are due to the isotope Pu^{240}). The quantity ν_{eff} is apparently somewhat low for the resonance at $E_0 = 26.1$ ev while Γ_γ is high because of the effect of the close-lying cadmium resonance (cadmium is used as filter for absorption of recycled neutrons).

SUMMARY

The values of the quantities Γ_f and Γ_γ are very sensitive to inaccuracies in the normalized results for $\nu_{eff\,T}$ for the thermal spectrum; furthermore, as is apparent from the table, the total width Γ cannot be determined with high accuracy from a comparison of the data published at the present time. Nevertheless, certain conclusions may be drawn on the basis of the present results.

The fission widths Γ_f in the levels of Pu^{239} differ markedly between themselves and may be grouped about levels ~20 mv and ~60-80 mv which is not inconsistent with theoretical predictions.

However, one should keep in mind the small number of levels which have been investigated and the possibility of omission of weak levels. In this connection the authors have paid attention to the fact that Farley [16] found a different exponential distribution of the fission widths in Pu^{239}.

The data of the present work indicate that the radiative width Γ_γ is not strictly constant but changes noticeably from level to level. This behavior of the quantity Γ_γ requires further investigation since marked changes of the radiative widths between levels have not been noticed before.

Considering self-shielding effects in the resonances and the contribution of the nonresonance term in the cross sections for absorption and fission (normally ascribed to the effect of remote "negative levels") it may be concluded that the data of the present work are in agreement with the results of other work on the determination of the energy dependence of ν_{eff} in Pu^{239} [17, 18].

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MECHANISM OF THE REDUCTION OF ZIRCONIUM AND TITANIUM OXIDES BY CALCIUM HYDRIDE

G. A. Meerson and O. P. Kolchin

The theoretical principles of the reduction of stable metallic oxides by calcium hydride are considered, on the basis of a study of the reduction reactions of zirconium and titanium oxides. Thermodynamical calculations and special experiments intended to clarify the mechanism of reactions between oxides and CaH_2 are used to compare the reducing activity of CaH_2 with those of the calcium and atomic hydrogen formed in its thermal dissociation. The view which exists in the literature, according to which the high reducing power of CaH_2 is explained by the action of the atomic hydrogen formed, is shown to be erroneous. It is shown that the principal reducing action in this process is effected by metallic calcium, formed by dissociation of CaH_2. Metallic calcium at temperatures of 1000° and higher reacts with chemically stable oxides not only in the liquid but also in the vapor state, which creates favorable conditions for rapid and complete reduction reactions. It is also shown that the active hydrogen liberated when metals saturated with hydrogen are heated is capable, to some extent, of additionally reducing the oxides which contaminate these metals.

Among the various methods for the preparation of such active metals as zirconium, uranium, titanium, etc., which have a high affinity for oxygen, considerable attention is devoted in the literature to the reduction of oxides of these metals by calcium hydride [1-5].

The opinion is expressed in a number of sources [1, 4, 5] that calcium hydride is a better reducing agent for chemically stable oxides even than calcium, being a more vigorous reducing agent because of the action of the atomic hydrogen liberated when calcium hydride is heated.

A characteristic of this method is that powdered hydrides and not the pure metals are obtained. Powdered hydrides are oxidized less than pure metallic powders in aqueous and acid treatments. Therein lies their advantage as starting materials for the production of compact metals by the methods of powder metallurgy.

The differences in the behavior of powdered hydrides and powdered metals on sintering, demonstrated by Hausner et al. [6] in the production of compact malleable zirconium by a metalloceramic method, are also worthy of attention.

Hausner et al. [6] showed that the vacuum sintering of a briquet pressed from powdered zirconium hydride proceeds more actively than the sintering of metallic zirconium powder. Sintering of the hydride yielded compact metal of higher density, more plastic, and of greater corrosion resistance. These differences were tentatively attributed to a lower content of nitrogen and oxygen impurities in the hydride in comparison with the powdered metal, and also to the fact that during sintering of the hydride, because of its dissociation in vacuum, contacts between the particles at fresh, active metallic surfaces were formed.

Although Hausner's experiments refer to the use of a hydride powder obtained by hydrogenation of compact metallic zirconium (wastes, scrap, etc.), nevertheless the results of these investigations may be extended to the possible use, in powder metallurgy, of zirconium hydride (and also of its analogs such as titanium hydride) obtained by direct reduction of the oxide by calcium hydride.

Moreover, in some cases hydrides of zirconium and similar metals have direct technical applications, for example in reduction in the production of certain alloys, in the welding of nonmetallic materials to each other and to metals (in particular, in the welding of silicon components in semiconductor technology), as sources of very pure hydrogen in some electrovacuum apparatus, etc.

Thus, hydrides can be used both instead of pure metals, usually obtained by reduction from the oxides by calcium, and for direct use in special cases.

Consequently, the choice between calcium and calcium oxide as reducing agent for the oxide of a given metal is sometimes dictated by the conditions in which the reduced product is to be used, and sometimes only by some particular advantages of the chosen reduction method.

In descriptions of reduction methods for chemically stable oxides by means of calcium or calcium hydride, mainly the technological conditions are usually compared, and much less attention is paid to the physicochemical nature of the reduction process with the use of calcium hydride in comparison with the use of calcium for reduction.

Nevertheless, for the correct choice of a technological method and of the direction of work for its further improvement a thorough study of its physicochemical nature is essential.

The present paper contains an examination of the physicochemical conditions of the use of calcium hydride as reducing agent for zirconium and titanium oxides, and discusses the controversial question of the possible advantage of calcium hydride as a more vigorous reducing agent than calcium because of the liberation of atomic hydrogen in the decomposition of calcium hydride.

To solve the question of the actual extent to which atomic hydrogen participates in the reduction, two fundamental factors must be elucidated:

1) the thermodynamic conditions of the reactions of atomic hydrogen and of calcium with the oxide undergoing reduction, and 2) the possible actual concentrations of atomic hydrogen which can react with the oxide.

We first consider the thermodynamics of these processes.

Thermodynamics of the Reduction of Zirconium and Titanium Oxides

The reduction of zirconium and titanium oxides by calcium hydride yields not the pure metals, but their hydrides or solid solutions of hydrogen in the metals, with dissolved hydrogen concentrations corresponding to the reduction temperature and hydrogen pressure.

In practice the reduction temperatures of different oxides may differ, but to standardize the calculation method and to make the results more comparable we shall assume the reduction temperature to be 1000°; this will not affect the theoretical deductions drawn from the calculated results in relation to other temperatures and oxides.

At 1000° C and atmospheric pressure the content of dissolved hydrogen in zirconium is 78 cm^3/g (i.e., corresponding approximately to the formula $ZrH_{0.05}$) and in titanium, 63 cm^3/g ($TiH_{0.27}$).

The aggregate reaction of the reduction of, say, zirconium dioxide by calcium hydride can be represented as

$$ZrO_2 + 2CaH_2 \overset{1000^\circ C}{\rightleftarrows} ZrH_{0.44} + 2CaO + 1.68H_2. \qquad (I)$$

This aggregate reaction may include the following particular reactions:

$$CaH_2 \rightleftarrows Ca + 2H, \qquad (II)$$

$$ZrO_2 + 4H \rightleftarrows Zr + 2H_2O, \qquad (III)$$

$$H_2O + Ca \rightleftarrows CaO + H_2. \qquad (IV)$$

$$Zr + 0.32H_2 \rightleftarrows ZrH_{0.64}. \quad (V)$$

$$ZrO_2 + 2Ca \rightleftarrows Zr + 2CaO, \quad (VI)$$

$$Zr + 0.64H \rightleftarrows ZrH_{0.64}. \quad (VII)$$

$$2H \rightleftarrows H_2. \quad (VIII)$$

Thus, the aggregate Reaction (I) can consist of the following stages:

Variant A (reduction of ZrO_2 by atomic hydrogen):

$$2(II) + (III) + 2(IV) + (V);$$

Variant B (reduction of ZrO_2 by calcium):

$$2(II) + (VI) + (VII) + 1.865(VIII).$$

Table 1 shows calculated values of the corresponding changes of free energy in these reactions for the reduction of zirconium and titanium oxides in standard conditions at 1000° C.

TABLE 1

Values of Free Energy Change at 1000° C in the Reactions Shown in the Table

Reaction number	Reaction	Value of $\Delta F^0_{1000^\circ C}$, cal
I'	$ZrO_2 + 2CaH_2 = ZrH_{0.328} + 2CaO + 1.6825H_2$ *	−66132
I"	$TiO_2 + 2CaH_2 = TiH_{0.27} + 2CaO + 1.865H_2$ *	−118341
II	$CaH_2 = Ca_{\text{г}} + 2H$	+70341
III'	$ZrO_2 + 4H = Zr + 2H_2O$	−184581
III"	$TiO_2 + 4H = Ti + 2H_2O$	−72158
IV	$H_2O + Ca_{\text{г}} = CaO + H_2$	−78241
V'	$Zr + 0.3175H_2 = ZrH_{0.635}$	−31874
V"	$Ti + 0.135H_2 = TiH_{0.27}$	−39383
VI'	$ZrO_2 + 2Ca_{\text{г}} = Zr + 2CaO$	−33378
VI"	$TiO_2 + 2Ca_{\text{г}} = Ti + 2CaO$	−87078
VII'	$Zr + 0.635H = ZrH_{0.635}$	−54347
VII"	$Ti + 0.27H = TiH_{0.27}$	−39938
VIII	$2H = H_2$	−70781

* The free energy changes in the reduction reactions of zirconium and titanium dioxides to give the metals or hydrides were calculated without intermediate reactions such as reduction of TiO_2 through TiO being taken into account, as they do not affect the general conclusions drawn from the calculated results.

The free energies of formation of the compounds involved in these reactions were first determined [7].

The $\Delta F^0_{1000^\circ C}$ values for the individual reactions given in Table 1 show that the free energy decrease in reduction of oxides by atomic hydrogen (III) is less than in the analogous reactions of the reduction of oxides by calcium (VI).

In calculations of ΔF^0_T for reactions (III' and III") it was assumed that the initial pressure of atomic hydrogen is 1 atmosphere. However, the equilibrium partial pressure of atomic hydrogen in the dissociation of calcium hydride, calculated from the equilibrium constant for this reaction at 1000° C, is only $1 \cdot 10^{-6,06}$ atmos while, for example, in the reduction of zirconium dioxide by atomic hydrogen its equilibrium partial pressure at 1000° C should be $1 \cdot 10^{-4,63}$ atmos.

If these reactions are considered at the atomic hydrogen concentrations which are the equilibrium values for CaH_2 dissociation, it is also necessary, in addition to the foregoing, to take into account that as the result of decomposition of CaH_2 to the equilibrium state (in which molecular hydrogen is also involved), a solution of hydrogen in calcium is formed, over which the partial pressure of atomic hydrogen liberated from CaH_2 should, under equilibrium conditions, be equal to the partial pressure of the atomic hydrogen formed by dissociation of molecular hydrogen. Therefore the reducing activity of atomic hydrogen in the system Ca−H must be equal in these conditions to the

reducing activity of molecular hydrogen. In other words, reduction of chemically stable oxides to the metals by atomic hydrogen the partial pressure of which is equal to the equilibrium value for the system CaH_2 (at a total pressure of 1 atmos) will not occur at all, as the free energy change in this case is positive.

In the real conditions of rapid decomposition of calcium hydride at 1000°C the partial pressure of atomic hydrogen near the surface of the CaH_2 particles will probably be above the equilibrium value.

In the presence of an appreciable concentration of atomic hydrogen, the reduction of a chemically stable oxide could to some extent be effected also by atomic hydrogen, especially if the influence of the calcium which decomposes the water vapor formed in the process is taken into account. Therefore, the possible participation of atomic hydrogen in the reduction process is primarily determined by the magnitude of its possible real concentration.

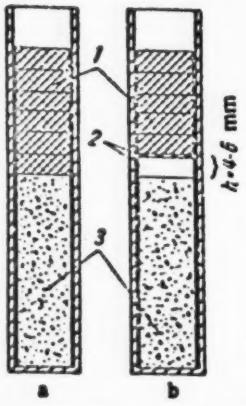
At atmospheric pressure and 1000°C the free path of a hydrogen atom before the first collision with a similar atom is negligibly small, and therefore, the atomic hydrogen concentration will have appreciable values practically only at the surfaces of the decomposing calcium hydride crystals. Thus, the surfaces of the refractory oxide particles, separated from the calcium hydride particles by pores and inter-particle spaces, will be reached by negligibly small concentrations of atomic hydrogen, and therefore the free energy decrease in the reactions of reduction of refractory metal oxides by atomic hydrogen, at the existing concentrations in the conditions of calcium hydride dissociation, will be immeasurably less than in reaction (III).

Hence it follows that the reduction of zirconium and titanium oxides by atomic hydrogen (by Variant A) is less probable from the energy aspect than reduction by calcium (Variant B), and the use of calcium hydride in this instance offers no thermodynamic advantages (contrary to the former view) over direct reduction by calcium.

In all the above discussions and calculations it was assumed that the calcium is in the liquid state, and the fact that at 1000°C the vapor pressure of calcium is 12 mm Hg was ignored. This assumption does not introduce any error into the above thermodynamical calculations, as the calcium vapor in the closed reaction space is in equilibrium with liquid calcium and therefore has the same free energy reserve as liquid calcium. However, the presence of an appreciable concentration of calcium vapor should have a favorable effect on the kinetic conditions for the reaction: the calcium vapor penetrating into the pores of the briquet, acts upon the whole surface of the oxide particles, and thus ensures that the reaction proceeds rapidly and completely.

Experiments to Study the Reaction Mechanism

To test the validity of the above theoretical considerations, certain experiments were carried out; these were not intended to solve technological problems and their purpose was to determine the mechanism of the reactions of zirconium dioxide and titanium dioxide with calcium hydride, but they made it possible to compare the roles of calcium and atomic hydrogen in these reactions.



Charging of the cases for the reduction of zirconium or titanium dioxide by calcium hydride. a) Direct reduction of oxides by calcium or calcium hydride; b) reduction by calcium vapor: 1) oxide; 2) support; 3) calcium or its hydride.

The powdered oxides were pressed into cylindrical briquets in the form of separate tablets. These briquets were placed into a cylindrical steel case hermetically welded at the lower end (with a steel plug at the top end) on top of a briquet of calcium hydride or metallic calcium, which was 150% of the amount theoretically required for complete reduction of all the oxide briquets. The arrangement of the briquets in the case is shown in Figure (a).

The case with the briquets was placed in the steel tube of a vertical electric furnace and kept there for 1 hour in an atmosphere of purified hydrogen or argon at 1100-1150°. After cooling in the same medium, each briquet was powdered separately, treated with dilute acid and distilled water to remove calcium compounds, and dried. Samples from definite layers (briquets) were then taken and analyzed for the zirconium (or titanium) and hydrogen contents in each layer.

In experiments in which the zirconium or titanium oxide briquets were in direct contact with the calcium hydride briquet in a pure hydrogen atmosphere it was found that the oxide briquets were reduced to a height of approximately 16 mm from their surface of contact with the calcium hydride briquet (Table 2).

TABLE 2

Results of Reduction of Zirconium (and Titanium) Oxides in Direct Contact With Calcium Hydride Briquets

Oxide	Distance from middle of each briquet of oxide to surface of contact with calcium hydride, mm	Contents in sample washed free from calcium, %			Total height of reduced layer, mm
		zirconium (or titanium)	hydrogen	sum of zirconium (or titanium) and hydrogen	
Zirconium dioxide	2	98.0	1.05	99.05	~16
	6	97.65	1.08	98.73	
	10	96.9	1.11	98.01	
	14	95.2	2.00	97.20	
	18	85.0	–	–	
Titanium dioxide	2	95.5	3.65	99.15	~16
	6	95.0	–	–	
	10	94.5	3.97	98.47	
	14	94.5	3.54	98.04	
	18	85.5	–	–	

As Table 2 shows, the sum of the contents of Zr + H or Ti + H in the reduced portions of the briquets reaches 99% (after removal of calcium oxide by acid).

The pores of the reduced portions of the briquets were found to be impregnated, in addition to calcium oxide (reaction product), also with excess calcium hydride, as shown by the evolution of hydrogen when the product was treated with water. Consequently, during the reduction the fused calcium formed by dissociation of calcium hydride was absorbed in the briquet pores by capillary forces, and was then hydrogenated while the briquet cooled in hydrogen.

To exclude the action of hydrogen liberated in the decomposition of calcium hydride, experiments similar to those described above were repeated with the same oxides, but with metallic calcium instead of its hydride and in an atmosphere of pure argon instead of hydrogen (Table 3).

It was found that in these experiments the reduction was again complete in all the parts of the briquet impregnated with calcium (to approximately the same height as in the previous experiments), and in the case of zirconium the results were even somewhat better and more consistent than with the use of calcium hydride in an hydrogen atmosphere.

As is seen from Table 3, the zirconium content of the briquets in the zone of influence of the calcium (i.e., to a height of up to about 16 mm from the surface of contact with the calcium briquet) is, within the limits of analytical accuracy, close to 100% (99.5-100%).

The quality of the titanium reduced under these conditions in an atmosphere of argon (95-99%) is worse than the quality of the titanium hydride reduced in a hydrogen atmosphere (98-99% total Ti + H). The reason for the difference is that metallic titanium powder is more strongly oxidized than titanium hydride when washed with with acid to remove calcium.

The purpose of the next series of experiments was to exclude direct contact between liquid calcium and the briquets of the oxide undergoing reduction. For this, zirconium oxide briquets were placed into a vertical steel case on a wire support at a distance of 4-6 mm above the surface of the calcium briquet (see figure, b). In this case the oxide briquets were acted upon only by calcium vapor.

TABLE 3

Results of Reduction of Zirconium (and Titanium) Oxides in Direct Contact With Calcium Briquets (in Argon)

Oxide	Distance from middle of oxide briquet to surface of contact with calcium, mm	Contents of zirconium (or titanium), %	Total height of reduced layer, mm
Zirconium dioxide	2 6 10 14.0 17.5	100.05 100.00 99.80 99.50 86.0	~16
Titanium dioxide	2.1 6.3 11.1 16.1	94.80 94.90 95.60 97.90	~18.5
Titanium dioxide	1.6 4.8 8.5 12.7	97.50 98.80 95.30 71.60	~10.6

TABLE 4

Results of Reduction of Zirconium Dioxide by Calcium Vapor (in Argon).

Height of gap between calcium and oxide briquets, mm	Distance from middle of oxide briquet to calcium surface, mm	Zirconium content in sample, %	Height of reduced layer, mm
4--6	4.8 7.9 12.5 16.8 20.7	99.95 99.60 88.0 74.8 75.0	~10.2
6	7.0 10.4 14.6 18.8	100.0 98.8 75.0 75.0	~12.6

These experiments showed that zirconium dioxide briquets were reduced by calcium vapor in an argon atmosphere to approximately the same distance from the calcium surface (see Table 4) as in the previous experiments with direct contact between the oxide briquets and the calcium, and with the same high qualitative result (up to 100% Zr).

Therefore, even without contact with the fused calcium the oxide is actively reduced by the appreciable concentration of calcium vapor above 1000°C ($P_{Ca_{1000°C}}$ = 12 mm Hg).

TABLE 5

Reduction of Titanium and Zirconium Hydrides by "Their Own" Hydrogen

Hydride	Contents in original sample, %			Contents in refined product, %			Increase of the sum total of titanium (or zirconium) and hydrogen, %
	titanium (or zirconium)	hydrogen	sum of titanium (or zirconium) and hydrogen	titanium (or zirconium)	hydrogen	sum of titanium (or zirconium) and hydrogen	
Titanium hydride	94.35 94.33 93.52	2.34 1.12 2.78	96.69 95.45 96.30	96.10 94.02 96.09	1.93 3.00 1.77	98.03 97.08 97.86	+1.39 +1.63 +1.56
Zirconium hydride	95.80 95.80 95.07	0.94 0.94 1.74	96.74 96.74 96.81	96.75 96.75 96.16	1.70 1.76 1.70	98.45 98.51 97.86	+1.71 +1.77 +1.05

Finally, in the last series of experiments an attempt was made to determine the reducing activity of at atomic hydrogen liberated by dissociation of the hydrides in absence of calcium. Tests were made of the possibility of additional reduction of zirconium and titanium hydrides contaminated with oxygen (i.e., oxides) by "their own" hydrogen.

The powdered hydrides containing 96-97% of the element and hydrogen together (the rest was oxygen) were made into briquets, additionally saturated with hydrogen for 30-60 minutes at about 500°, and then heated in pure and dry hydrogen at 1100-1150°, when some of the hydrogen, because of its reduced solubility at this temperature, could be liberated from the hydrides in the atomic state.

The briquets were cooled in a hydrogen atmosphere. Analyses of the hydrides treated in this way showed an increase of 1.0-1.7% in the sum of the element and hydrogen, but nevertheless, in this condition the reduction was not complete (see Table 5). Consequently, partial additional reduction of the oxide films on the surface of the hydride grains took place by active hydrogen liberated from the hydrides themselves.

In addition, experiments were performed on the reduction of zirconium dioxide by atomic hydrogen liberated in the dissociation of tantalum hydride. The method used for these experiments was similar to that described above (see figure, a).

The zirconium dioxide briquets were placed in a vertical steel case lined with sheet tantalum, on top of a briquet of tantalum powder previously saturated with hydrogen (one volume of tantalum absorbs up to 700 volumes of hydrogen). The cases were held in a hydrogen atmosphere at 1100°. However, zirconium dioxide was not reduced in these conditions.

It is probable that the atomic hydrogen liberated from tantalum hydride outside the zirconium dioxide briquet recombines rapidly into molecular hydrogen at the tantalum surface and therefore it exerts practically no reducing effect on the oxide briquet.

Another difference between these and the preceding experiments is that, as the result of decomposition of the hydrogen-saturated titanium or zirconium hydride, hydrogen is liberated throughout the thickness of the briquet, displacing water vapor (reaction product) from the briquet pores and so shifting the reaction in the direction of reduction.

SUMMARY

1. Thermodynamic calculations for the reduction reactions of the oxides of zirconium and titanium and special experiments were used to compare the reducing activities of calcium hydride and of the calcium and atomic hydrogen liberated in its thermal dissociation.

It is shown that the free energy decrease in the reactions between atomic hydrogen and zirconium and titanium dioxides is less than in their reduction by calcium in an atmosphere of molecular hydrogen, even if it is assumed that atomic hydrogen can exist in a metastable state at a partial pressure of one atmosphere. In real conditions of the process the partial pressure of atomic hydrogen is negligibly low, which sharply diminishes its reducing activity.

This shows that the existing view that the high reducing activity of calcium hydride is to be explained mainly by the action of the atomic hydrogen liberated is erroneous.

2. Calcium reacts with chemically stable oxides above 1000°C not only in the liquid but also in the vapor state, which creates favorable conditions for acceleration of the process and for complete reaction of the calcium with particles of the oxides undergoing reduction.

3. The active hydrogen liberated in the dissociation of hydrides of refractory metals at temperatures above 1000°C can to some extent reduce the thin oxide films on the surfaces of crystals of these metals (or hydrides).

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RADIOACTIVE CONTAMINATION OF THE ATMOSPHERE

L. I. Gedeonov

This article is a review of foreign works on the study of radioactive contamination of the atmosphere due primarily to diffusion of fission fragments from atomic tests. Several properties of fission fragment activity, namely its decay rate and the composition of its radiation, are considered, and methods are described for monitoring the fallout from the atmosphere and calculating the resulting radiation dose. The dose is almost everywhere much less than the permissible limit. At several places, however, fallout densities and activity concentrations are observed which are close to the allowable limit. A method for constructing the trajectories of air masses is briefly described, and it is shown that radioactive clouds migrate over the whole world, producing a radioactive layer in the atmosphere at an altitude of 8-12 km consisting primarily of the long-lived nuclides Sr^{90}, Ce^{144}, and Pu^{239}. Of the fission fragments contaminating the earth's surface the most dangerous is Sr^{90} which has the property that it accumulates in the organisms of domesticated animals and thus enters the human organism. The radiation level at points far from the test areas is at present far below the allowable limit in the sense of the possible occurrence of somatic diseases, although even at the present radiation level, effects on posterity cannot be excluded.

When atomic and thermonuclear bombs are exploded a large amount of radioactive matter enters the atmosphere. Lapp [4] and Sittkus [5] assert that several days after an explosion the amount of this radioactivity can be measured in millions of curies. According to an evaluation performed by Rottblatt [6], in one case the activity of substances in the atmosphere eight hours after an explosion was 10^{11}-10^{12} curies.

When an atomic bomb of average size explodes, the total activity of the fission products is 8.2×10^{11} curies one minute after the explosion, and 2.3×10^{6} curies at the end of the first month [7].

The radioactive dust rises to an altitude greater than 30 km, after which radioactive particles begin gradually to settle out. If the density of the substances of which the dust particles is composed is taken as 3, the average precipitation rate of particles with a diameter of 100 μ is approximately 60 m/min [8]. Therefore, gravitational precipitation may play a dominating role in settling out relatively large particles. Particles of the same density, but with a diameter of 1 μ precipitate at the rate of 0.6 cm/min. For such slow precipitation of small particles, the dust may be carried by air current to practically any point on the earth, perhaps even circling it more than once. Therefore the effects of radiation emitted by fission products may appear anywhere on the earth.

If we assume that at some time the total activity of the fission products is 5×10^{6} curies and that the radioactive substances are distributed uniformly over the whole globe in a 10 km thick air layer, the radioactive concentration in the air should be about 10^{-12} $\mu c/cm^3$ [5]. The natural radioactivity of the air due to radon and its daughter elements is on the average 10^{-1} $\mu c/cm^3$. Comparison of these numbers would seem to lead to the reassuring conclusion that the additional radiation due to fission products is of no significance. It should, however, be noted that it is possible to accumulate long-lived nuclides which are extremely harmful biologically. Furthermore, the distribution of radioactive contamination over the earth's surface is not actually uniform, and one may not exclude the possibility that at some point there will appear a sufficiently high concentration of radioactive fission fragments [9-13].

Radioactive contamination of the atmosphere must be taken into account when measuring the radioactivity of substances, when prospecting for radioactive minerals, when preparing photographic emulsions, and so on. Soon after the world's first atomic explosion in 1945, Webb [13] reported the unusual radioactive contamination of a carton used for packing photographic materials, and established the fission-fragment nature of this contamination.

At the present time observations are performed everywhere [7, 14-19]. Special bureaus have been organized in the U.S.A. and Japan to monitor the atmospheric radioactivity [4, 9, 10, 15-17]; in France [18], Germany [5, 19], Italy [20], and Canada [21] many laboratories devote time to the investigation of these phenomena. A network of observation points has also been organized by the Academy of Sciences, USSR.

Artificial radioactivity is an excellent method for labelling air masses. Many meterologists and aerologists have made use of this fact. In recent years the study of natural atmospheric radioactivity has been significantly revived [18-34]. The necessity for performing measurements of very weakly radioactive substances provided new impetus to the development of fine and extremely sensitive methods for the solution of problems related to the observation of atmospheric radioactivity [1, 7, 14, 22, 35-41].

In studying artificial atmospheric radioactivity, the following main directions are being taken:

The gross fission product radioactivity is being studied with extreme care; the following points are being examined: the composition and energy of the radiation, the type of radioactive decay, and the contribution of the separate nuclides to the total radioactivity.

Constant daily observations are taken of the sea-level density of radioactive fallout so as to determine the present radiation dose of the population [5, 8, 9, 10, 12, 15-17], which is necessary, in particular, to determine the possible genetic effects of radioactive atmospheric contamination [42-44].

In order to determine the dose which can be obtained by breathing radioactive dust, the concentration of radioactivity in the air is systematically investigated [5, 7, 8, 15, 45-47].

Meterological observations are being performed on the movements of labeled air masses, and the possible effect of explosions and radioactive contamination on the weather is being studied [5, 14, 17, 19, 29, 48-52].

Radiochemical investigations of the composition of aerosol are being performed so as to study the separation of various elements as they travel through the atmosphere [9, 10, 24, 27, 53], since this is an essentially new geochemical tool, namely the application of labeled atoms to the investigation of geochemical processes.

1. The Gross Fission Product Radioactivity

The radioactive properties of the gross fission product of U^{235} have been investigated by Way and Wigner [54]. The decay rate of the gross product decreases constantly, since the short-lived elements gradually "drop out of the game." To a sufficient degree of approximation, one may set up the formula

$$A_t = A_1 \cdot t^{-\gamma},$$

where A_t is the radioactivity of the gross product at a time t after fission, A_1 is the activity at a unit time after fission, and γ depends on the composition of the mixture investigated. Way and Wigner give $\gamma = 1.2$ for the gross fission product. When fragments are collected from the atmosphere, there may have taken place a separation along their path of flight or in the process of treating the specimens. Nevertheless the radioactivity of actual atmospheric specimens decreases according to this law [9, 10, 14, 19, 55]. Various values from 1 to 2.1 are given for γ and according to the observations of many authors the values they give will describe the decay of fission products of various ages and for long term observation. Other authors state that for different ages of the fission fragments, the decay is described by various values of γ. Chatterjee [55] proposes that the question of the value of γ be solved separately for each specimen, using the method of successive approximations.

For most of the atmospheric specimens, γ may be considered constant and close to 1.2. Sometimes γ is observed to decrease in time. This is explained by contamination of the specimen by long-lived nuclides. Radioactive clouds of various ages, migrating over a long period of time, gradually mix with each other. As a result there are always long-lived substances in the atmosphere, and these give a background radioactivity. Some

admixture of these substances to "fresh" samples will be negligibly small before the latter have aged, so that the decay will take place according to the Way-Wigner law with γ close to 1.2. As the activity of the "fresh" nuclides begins to decrease noticeably, the relative background increases, so that the decay rate decreases and is described by γ less than 1.2. In the case of higher decay rates with γ as large as 2.1, either a separation of the fission fragments was known to have taken place (for instance, in measuring the radioactivity of samples obtained by burning the skin of fishes caught in the Pacific Ocean [9, 10], or else the radioactivity included that due to the effects on the surrounding substances of neutrons produced in the explosion; this additional activity is due primarily to short-lived isotopes of several elements.

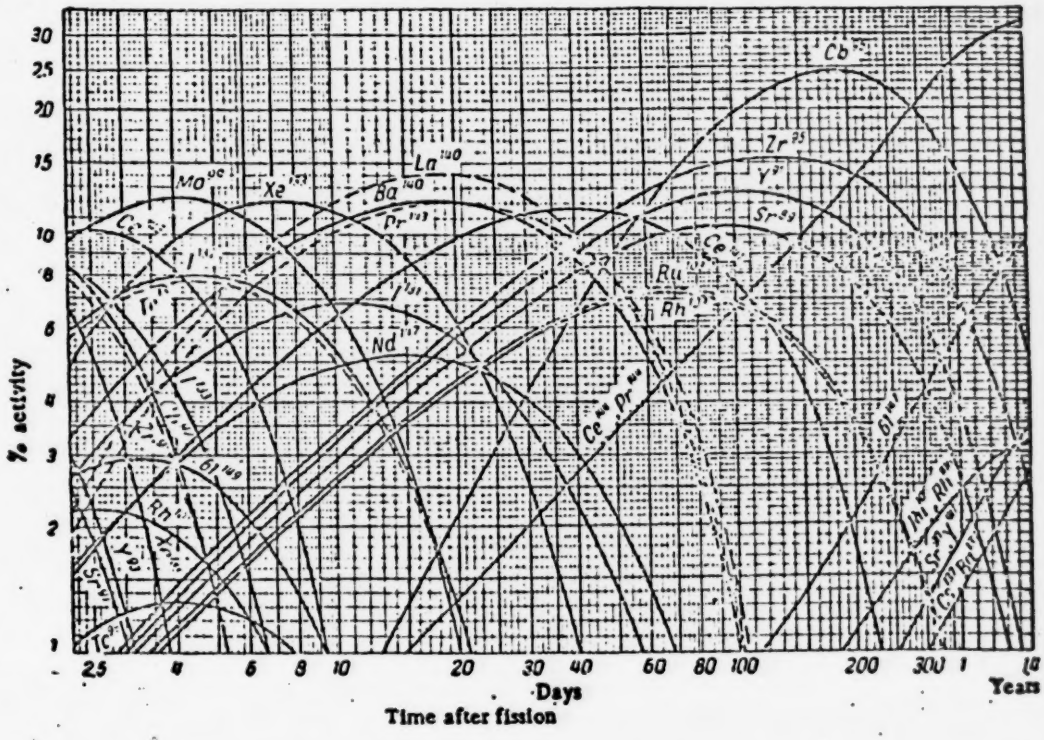

Fig. 1. Relative radioactivity of product nuclides in the gross fission product radioactivity of U^{235} (first 1.5 years).

It is often necessary to know the contribution to the total radioactivity from one or another nuclide. For this purpose Hunter and Ballou [56] performed a calculation based on the fission product yield of U^{235} and the half-lives of the fragments. The results of these calculations are shown in Figs. 1 and 2.

A similar calculation has been performed for the fission product radioactivity in reactors [57].

To perform absolute measurements and dosage evaluations, it is convenient to find substances whose γ and β-radiation is equivalent to that of the atmospheric samples. For this purpose absorption measurements have been performed for the γ and β-rays emitted by samples containing atmospheric activity. Figure 3 shows a typical absorption curve for β-radiation, obtained by Clark [49]. The sample was about two days old. Although the curve is typical for a complex mixture of emitters, it can be divided into three parts, whose average energies are about 0.8, 0.3, and 0.1 Mev. Wheler and co-workers [58] found that a mixture of fragments several days old has an average β-energy of about 0.95 Mev. As a standard for measuring the β-activity on the fission products, they recommend Tl^{204}. Sittkus [5] assumed that for the mixture of radioactive products

from the atmosphere, the counter efficiency is the same as for the β-radiation of potassium. A group of scientists from the Naniva University (Japan) [9] have performed a spectral analysis of the β-radiation of radioactive dust using a β-spectrometer. It was found that radiation with an energy lower than 0.4 Mev predominates.

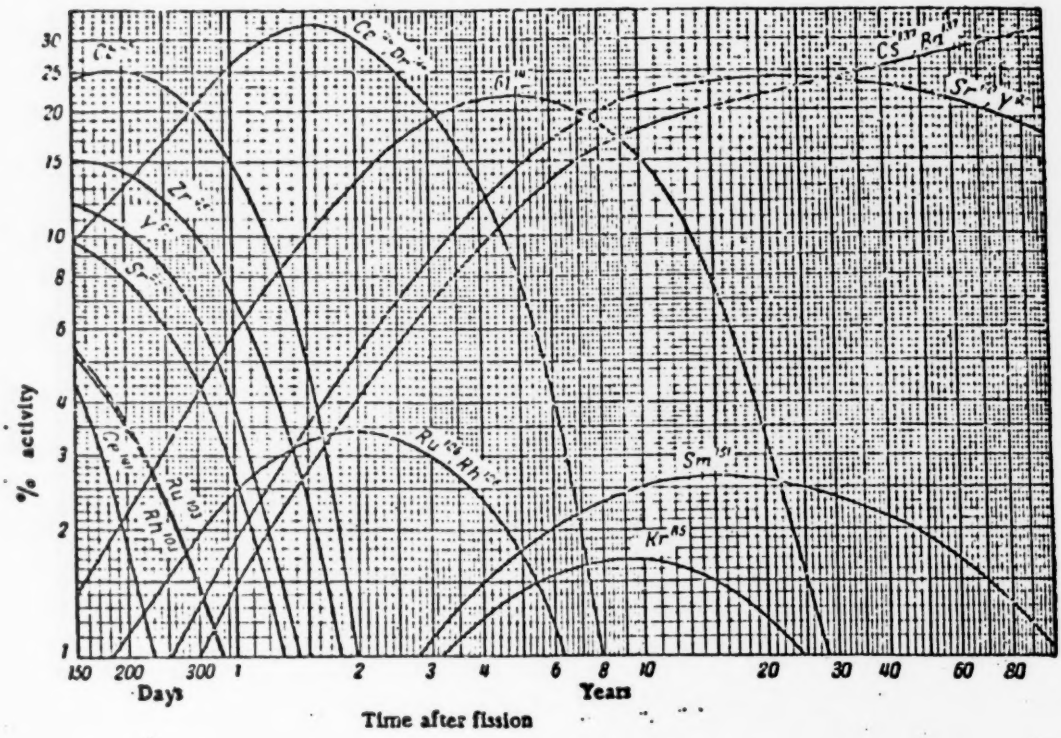

Fig. 2. Relative radioactivity of product nuclides in the gross fission product radioactivity of U^{235} (from 150 days to 80 years).

Literature on the gamma-radiation from the fission products is very scarce. In the communication from the American delegate to the U. N. Scientific Committee on the Effects of Nuclear Radiation [59] the mean gamma-ray energy is assumed to be 0.7 Mev.

The works mentioned here are separate and disconnected and it is therefore necessary to undertake a systematic study of the properties of fission products in the atmosphere.

2. Radioactive Fallout Density

The fallout of radioactive products from the upper layers of the atmosphere should be characterized by the surface density of the fallout at the earth's surface and the radioactive concentration in the lowest level of the atmosphere.

Every day some amount of radioactive matter is deposited on the earth's surface, most of it descending with atmospheric precipitation. Several methods for collecting these substances have been proposed [5, 8, 15, 18, 38, 59]. The most common is the collection of settling radioactive dust on adhesive paper or film. In the U.S.A. there exists a special bureau organized by the Health and Safety Laboratory of the A. E. C. [8, 15, 59].

There exist more than 100 observation points where adhesive film is exposed to the open air every day. The exposure time is 24 hours, after which the film is folded and sent to a laboratory in New York, where the investigation of the radioactivity is carried out. Here the film is incinerated and a sample is prepared of the ashes; no earlier than one day after the collection, when the natural radioactivity has decayed, the β-activity of this sample is measured. The measuring process is automatic. The radioactivity of daily fallout can be normalized to unit area, and thus its density may be determined. The fallout density is conveniently measured in units of $\mu\mu c/m^2$. The counting apparatus must first be calibrated in order to find the total number of decays from the number of count per minute of the sample. By knowing the total number of decays and the area of the film onto which the radioactive matter was deposited, the fallout density may be determined.

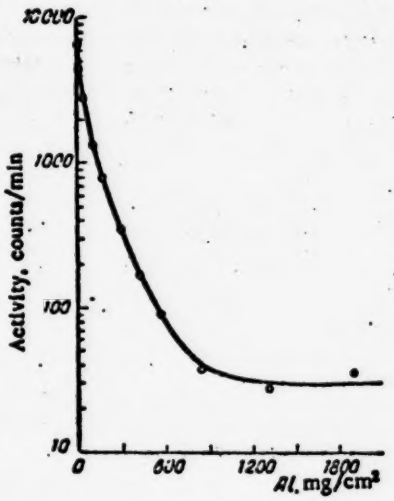

Fig. 3. Absorption curve for β-radiation from fission fragments (rain specimen).

In Japan the collection is accomplished in vessels containing some water [60]. The American delegate to the U. N. Scientific Committee on the Effects of Nuclear Radiation [59] recommends the use of deep containers with a small quantity of water. A comparison of this method of collection with the adhesive paper method showed that the water-container method is more efficient by a factor of 1.8.

By long and continuous observation of the fallout density it is possible to evaluate the amount of radioactivity accumulated at any place. This is done by taking account on the one hand of the daily radioactivity added, and on the other hand of the decrease in the radioactivity due to decay and the action of natural factors, such as soil drainage, evaporation, chemical exchange, etc. In actuality the calculation is based on the amount added and the decay, and the effect of the remaining factors has not yet been well studied. It is therefore of interest to calculate the accumulation on the basis of decay only, and then to compare the activity level of the soil as predicted by this calculation with its actual activity. This comparison is convenient to make for a single artificially created nuclide. Since Sr^{90} is biologically the most dangerous one, it is the most important one to determine. The American bureau has performed such determinations [8, 15, 59, 61]. Figure 4 shows the results of calculation for the total accumulated radioactivity density due to fission products averaged over the northeastern region of the U.S.A. The same figure shows the Sr^{90} content calculated for a unit area of soil. From 1951 to 1955 the total activity accumulated on the earth's surface has increased significantly. The calculated Sr^{90}

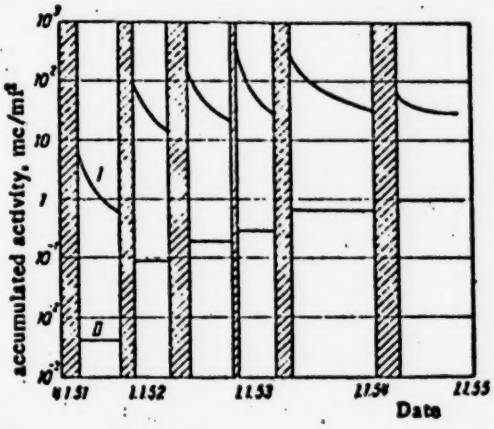

Fig. 4. The accumulation of fission products in the northeastern United States, as given by experiments performed from the beginning of 1951 to January 1, 1956.
The cross-hatched regions correspond to periods during which series of tests were carried out in the U.S.A.
I) total radioactivity; II) Sr^{90}.

content in the soil has increased by a factor of more than 100. According to the data from 17 stations in the U.S.A., it is found that the Sr^{90} content is higher on the average than that given by calculation.

On the basis of data on the accumulated radioactivity of the fission products, we can calculate the γ-radiation dose to which an organism is subjected if continuously exposed at a contaminated point. To do this, it is necessary only to establish the relation between the surface density of the total fission fragment radioactivity and the γ-ray dose intensity due to this contamination. The first calculations of this relation were undertaken by Thirring [62]. There exist also experimental results for determining the γ-ray dose intensity due to a given radioactivity surface density [15]. For instance, for a contamination density of 120 mc/mi^2 the γ-ray dose intensity is 10^{-6} rad/hr.* In more convenient units, the proportionality factor becomes $21 \frac{\text{rad}}{\text{hr}} / \frac{\text{curie}}{\text{mi}^2}$. On the basis of other data [63], however, this factor is only 10.8 $\frac{\text{rad}}{\text{hr}} / \frac{\text{curie}}{\text{m}^2}$. Such disagreement between the data in the literature indicates the necessity for more accurate experiments in order to clarify the dependence of the dose intensity on the fallout density.

It is accepted practice to determine the danger due to radioactive contamination of the soil from the γ-radiation alone. This is because it is assumed that humans do not spend any significant time in direct contact with the soil, and that footwear and clothing absorb any β-radiation.

When the dose intensity is determined, the total dose for infinite exposure is not difficult to calculate. The dose intensity decreases in time together with the activity according to the Way-Wigner equation. Integration of this expression over time will give the total dose. The results of such a determination are shown in the graph (Fig. 5) [4], where 100% is taken as the dose for infinite exposure at a contaminated place; the doses obtained for other exposure durations can be found from the graph. It can be seen that a large part of the total dose, about 50%, is obtained during the first days, and that 73% of the total is obtained during the first month of exposure in the contaminated region.

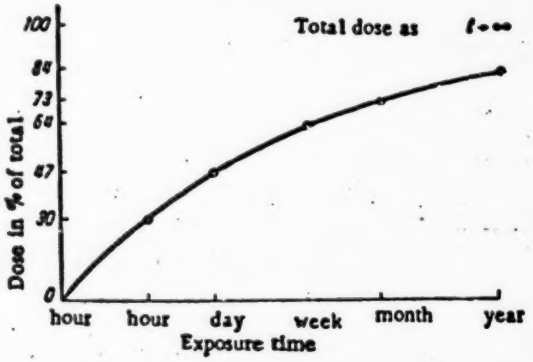

Fig. 5. The radiation dose as a function of the exposure time in a region contaminated by fission fragment activity.

The absolute value of the radiation dose seldom attains a value somatically dangerous in any degree. As a rule, the fallout which is observed gives a dose which varies between the limits of 0 and 100 mrad. Such a dose is far below the maximum allowable one.

In addition to surface radiation, radioactive substances settling out on the soil can be dangerous by entering into the organism. Cases of contamination have been observed for vegetables, fruit, water reservoirs, and grass in pastures [9, 10]. This situation requires particularly careful study, since food may become a source of human contamination.

* A rad is the unit of absorbed energy which corresponds to about 1.2 roentgens of radiation for human and animal tissue.

3. Radioactive Contamination in the Air

In addition to the biological effect of γ-rays from radioactive fission fragments settling onto the earth, radioactive dust may enter the organism also by breathing. It is known that a human inhales about 10 m^3 of air on the average in 24 hours. This volume of air may contain a significant amount of radioactive dust even in small concentrations, and this dust, entering the organism, may be dangerous. Therefore, even some time ago, several laboratories [5, 7, 10, 14, 15, 18-21] performed observations of the radioactivity concentration in the air. Various types of apparatus have been developed to collect dust by filters or other means [1, 35, 38, 64, 65].

The measurement of the radioactivity of samples so obtained is performed after the natural radioactive substances have decayed. The measuring apparatus is first calibrated so as to obtain the activity in curies. If the volume of air from which the dust is collected is known, one can calculate the radioactivity per unit volume, i.e., determine its concentration.

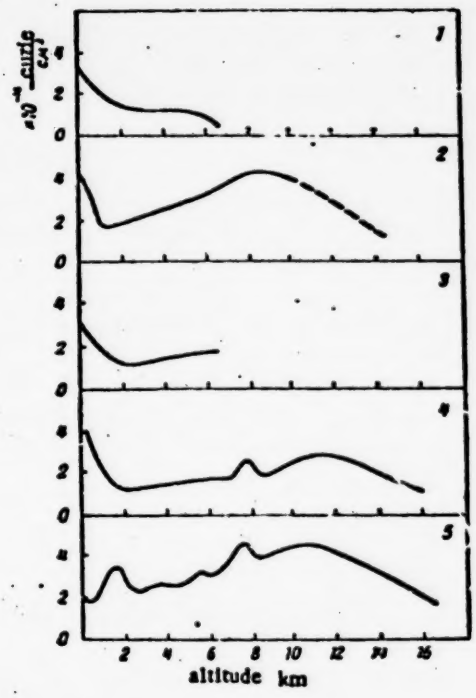

Fig. 6. Vertical distribution of radioactive substances in the atmosphere.
1) 13.08. 1954 Tateno (near Tokyo); 2) 23.12. 1954 (Tokyo); 3) 13.01 1955 (Tokyo); 4) 2.02. 1955 (Tokyo); 5) 22.02 1955 (Tokyo).

In determining the radioactive concentration in the air, it is important to use collecting methods whose efficiency is either 100% or can be accurately calculated. In the recommendations of the U.S.A. to the UN [59], it is stated that little is known as to the distribution in size of the artificially radioactive atmospheric dust particles collected at large distances from the test points. One can assert only that the dimensions of the radioactive dust particles are most often less than 5 μ. A quantitative study of the dimensions of particles less than 1μ has not yet been undertaken due to the technical difficulties. It may be assumed, however, that the dust contains many particles whose dimensions are less than 1 μ. It is thus important to collect particles whose dimensions are at least as low as 0.1 μ. At present many types of filters have been suggested which are capable of collecting such particles. The air intakes used are usually of the dust-collecting fan type [7, 8, 14, 15, 19, 59].

The determination of the radioactive dust concentration in the air is performed primarily at relatively low altitudes; i.e., that layer of air is studied which is breathed by living organisms. The results of many such determinations have been published [5, 8, 15, 18, 38, 46, 47, 64, 65]. During the tests in Nevada, U.S.A. in the Spring of 1952, the maximum radioactive contamination of the air was observed in Elko, Nevada, and was $23 \cdot 10^{-15}$ curies/m^3. According to Ishii [66], from August 1954 to February 1955 a concentration of the order of 10^{-10} curies/m^3 was observed in Tokyo, i.e., a concentration several thousands of times greater than in Elko. When the fallout density and radioactive concentration are continuously monitored, no correspondence is observed between the variation of these quantities.

In addition to determining the radioactive concentration in the lowest layer of the atmosphere, it is interesting to study the dependence of this concentration on altitude. The fission fragment distribution in the atmosphere is of interest not only in studying the effects of radioactivity on living organisms, but also in investigating the movement of air masses.

The opinion exists* that a continuous accumulation of fission fragments is taking place in the stratosphere, and that fragments of various ages are gradually becoming more and more intermixed. These substances partially settle out of the atmosphere onto one or another region of the earth. This assertion is somewhat moot. Several investigators believe [5, 10, 19] that the atmosphere is "cleaned" of fragments in a relatively short time, say in three or four months.

The investigation of the radioactive dust in the upper layers of the atmosphere is undertaken with the aid of radiosondes and aircraft [66]. Figure 6 presents the results of measurements in five counter-carrying balloon ascents in the neighborhood of Tokyo in 1954-1955. It was found that the maximum radioactivity is located near the earth and at an altitude of 8-12 km. Parallel experiments on the collection of dust by aircraft leads to the conclusion that almost all of the activity at 8-12 km is due to fission fragments.

4. Meteorological Observations

Investigation of the paths taken by the fission fragments, their velocity, their time in the atmosphere, etc., requires knowing in which explosion they were produced. This can be done by determining the age of the radioactive cloud, even if the time of the explosion is unknown. If the position of the explosion is known, its time can be determined from the time at which the shock wave arrives, which is registered as an anomalous maximum on a microbarograph [10, 60]. When, however, the explosions follow each other after intervals of several days, it is difficult to determine from this data which explosion produced the fragments collected at one or another observation point. Therefore the age of the fragments is determined better from their specific radioactivity. Since the decay rate of the fragments is known, observation of the decay of a sample can give its age. In principle, two measurements will suffice. In practice, a graphical method is used to determine the age from the measured data. By plotting the ordinate in terms of the quantity

$$y = \frac{1}{\sqrt{A_t}},$$

and the abscissa in terms of the data of measurement, a straight line graph can be obtained with an intercept on the abscissa corresponding to the test data. This method has been used by many authors [5, 14, 19, 50, 55]. The approximate value 1, rather than 1.2 as given by Way and Wigner [4], is sometimes used for γ. An example of a dating graph is shown in Fig. 7, [55].

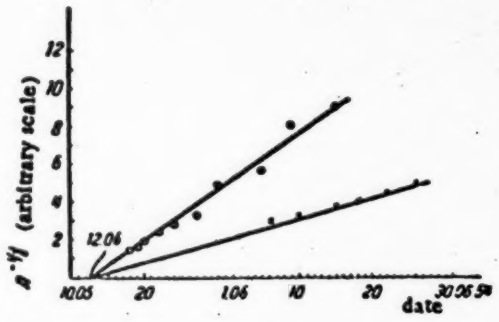

Fig. 7. The determination of the date of a nuclear explosion on the basis of the Way-Wigner formula (with $\gamma = 1.3$) from the decay of two atmospheric specimens collected in Calcutta.

* Described in the report of the English committee on the effects of nuclear radiation (July, 1956).

The age of the fragments at their arrival time determines their flight time from the explosion point to the observation point. If there is a network of observation points it is thus possible to follow the path of the fragments for long distances [17, 19, 48-50, 60, 67]. The radioactive clouds may pass over several points without causing any radioactive fallout. On the other hand, at any point the effect of the fragments may be measured several times. Fragments from a single given explosion may appear several times at various significant time intervals and in the intervals the effect of fragments due to other tests may be observed. Therefore the trajectory of the air masses from the test points is traced out on the basis of ordinary meteorological data, and is then compared with the results of observations of radioactive fallout [50, 60, 67, 68]. The construction of these trajectories is extremely interesting for the study of various meteorological factors which take part in the motion and precipitation of fallout. Figure 8 shows such a construction [68] for the motion of fission fragments from an explosion in the Las Vegas, Nevada, U.S.A. region on May 25, 1953.

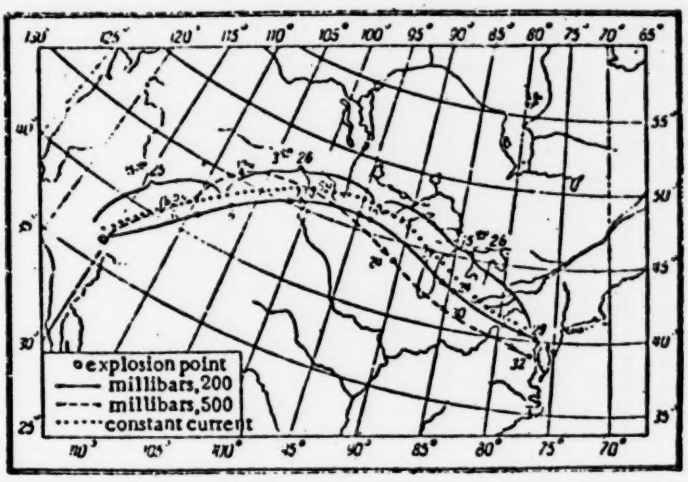

Fig. 8. The trajectory of air masses starting at 3:30 PM, May 25, 1953 from the explosion point in Nevada. The trajectories are traced along levels with pressures of 200 millibars and 500 millibars.
The approximate position of the line of constant air mass current is also shown.

In the process of studying the migration of radioactivity in the Northern Hemisphere, it was gradually seen that the radioactive clouds at high altitudes are moved by the prevailing constant air currents in a general easterly direction at an average velocity of 1000-2000 km per day [60]. Sometimes this velocity is as high as 3000 km per day. In its motion, the radioactive cloud sometimes breaks up, its motion is complicated by atmospheric perturbations, and gradual diffusion takes place. As a result, the effect of this radioactive cloud can be observed almost all over the hemisphere.

In consequence of the wide establishment of observation stations [7-9, 14, 15, 59], it was seen that the various fragments are often separated; this seems to be related to the various geochemical behaviors of the elements whose isotopes are among the fragments.

5. Radiochemical Analysis

A radiochemical analysis of atmospheric specimens is necessary to solve many problems.

The first problem consists of verifying whether or not the atmospheric radioactivity is due to fission fragments. In addition to fission fragments, radioactive nuclides are observed which are produced by the effects of radiation on the matter surrounding the explosion point. Thus, in Japan analysis of ash due to the Bikini tests of 1954 indicated the presence of induced activity of S^{35} and Ca^{45} [9, 10, 12, 45-47, 53].

Radiochemical analysis also resolves the question of the composition of the radioactivity from the point of view of its health hazard. Short-lived fragments are less hazardous when they enter the organism than are long-lived ones. It is also important to know whether specimens contain radioactive isotopes of those elements which can be assimilated without elimination, thus accumulating in the organism. In this respect, radioactive strontium is particularly dangerous. When it enters the organism, almost all of it remains in the bone tissue. Among the fission fragments are the isotopes Sr^{89} and Sr^{90}, the latter having a half-life of 18 years.

The third question is the separation of the fission fragments as they are carried through the atmosphere. It is interesting to study the composition of radioactive specimens from a single source collected at various points along the path of the radioactive cloud [8-10]. A difference in the composition of these specimens would give an indication of the separation along the line of flight, which is important for the geochemical study of the behavior of many elements.

At the present time there exists no study generalizing the results of work on radiochemical analysis.

The most complete study, from the radiochemical point of view, was that of the fallout in Japan in the spring of 1954 [9, 10, 53]. Among the radioactive nuclides were found most of the possible fission fragments. The quantitative ratios of the various nuclides in the total makeup of the radioactivity was determined. It was established that in older specimens the main part of the radioactivity is due to the nuclides Sr^{90} and Ce^{144}.

In samples about 25 days old, a significant role should be played by Ba^{140} and its daughter La^{140}, as well as Pr^{143}. This was actually observed, though the activity of Ba^{140} and La^{140} were much lower than that calculated. This decrease would seem to be only apparent and due to the fact that in calculating the weight of the barium carrier introduced in the radiochemical analysis, the weight of barium contained in the atmospheric dust was not taken into account, though it is present in measurable quantities. The radioactive dust often contains U^{237} which is produced in the explosion from U^{238} by the (n, 2n) reaction. In several cases the α-emitter Pu^{239} is observed which has not undergone fission in the explosion; together with Sr^{90}, this is one of the most dangerous components of artificial atmospheric radioactivity.

6. Biological Effects

The presently observed fallout density of radioactive fission fragments and their concentration in the air does not on the average reach a dangerous level. At several separated points, however, fallout has occurred which gives a dose extremely close to the maximum allowable limit. It often occurs that the radioactive concentration in rain water is greater than the maximum allowable limit [5, 60]. In this connection recommendations have been put forth for the purification of rain water from radioactive contamination [60].

The somatic effects of radioactivity on the organism are at present studied in sufficient detail under the name of radiation sickness. Up to the present time radiation sickness has been observed during times of peace in unfortunate cases during tests [9-11] and although these cases are isolated, the number of victims is already large; no cases of radiation sickness due to atmospheric radiation have been recorded at large distances from test areas.

There is, however, as yet no complete assurance that the effect of radioactive contamination of the atmosphere is harmless [42]. The concept of the maximum allowable dose itself is continuously undergoing criticism and reappraisal [42, 69, 70].

In recent years the question of the genetic aftereffects of radioactive fallout over the whole world has been actively discussed [4, 6, 42, 71]. A quantitative solution of this problem requires further observations and experimentation in the field of genetics. It cannot be doubted, however, that excess radiation obtained by an organism without harm to itself may cause serious irregularities in future generations, which it is claimed can have adverse effects on the species. Of all the fission products of uranium and plutonium, the most dangerous is the radioactive isotope Sr^{90}. Together with its long half-life (18 years), Sr^{90} easily enters the organism and becomes firmly established in the bones together with calcium.

In entering the organism of domesticated animals, in particular cows [72], a large part of the strontium is eliminated with milk and may thus enter the human organism with food. It should also be emphasized that according to the most recent investigations [73, 74], the Sr^{90} content in the soil has a tendency to increase.

According to the data presented in a review by Indian scientists [75], after the explosion of a powerful hydrogen bomb, the dose of radioactive strontium for each human being on the earth attains a quarter of the maximum allowable dose.

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LETTERS TO THE EDITOR

SOURCE OF MULTIPLY-CHARGED NITROGEN IONS FOR A CYCLOTRON

P. M. Morozov, B N. Makov, and M. S. Ioffe

In [1] there was a description of a cyclotron ion source which furnished triply-charged nitrogen ions. Below is presented a brief description of a cyclotron source which is capable of furnishing quintuply-charged nitrogen ions.

The strongest ionization of molecules by electron impact is found in an oscillating discharge, in a strong longitudinal magnetic field in which a high current-density of fast electrons is achieved. We have developed an ion source which makes use of this effect; the design of the source is shown in Fig. 1. It consists of a discharge chamber, an anode A, a cathode C with an indirect heater H and a reflector R. The discharge chamber has cross-sectional dimensions 10 × 10 mm and the length of the chamber is 100 mm.

The gas is admitted to the discharge chamber uniformly over the height, through a series of apertures drilled in the walls opposite the exit slit. The dimensions of the ion exit slit are 2 × 20 mm.

The chamber is made from graphite or red copper. In the latter case, the chamber is water-cooled and the wall E which contains the exit slit is made from molybdenum.

The indirectly-heated cathode consists of two main components: a tungsten parallelopiped C with cross-sectional dimensions 7 × 7 mm and 14 mm in length and a tungsten filament H, 1.2 mm in diameter. The tungsten parallelopiped is the working cathode for the arc discharge. It is heated by bombardment of electrons which are emitted from the tungsten filament and accelerated by a potential difference which is applied between the filament and the working cathode. The maximum power required for heating the cathode is 800 watts at an electron current from the filament of 0.8 amp and an accelerating voltage of 1,000 volts.

The filament is heated by an 80-amp alternating-current source at a frequency of 10^6 cps. To reduce the heater power in the working cathode when the discharge is fired, and to achieve a uniform emission over the surface of a cathode, thereby also achieving uniform evaporation, the side surfaces of the cathode are covered by a molybdenum shield S. The shield and the operating cathode are mounted on a molybdenum plate which is connected to the cooling water by a red-copper ring.

Opposite the cathode, at the other end of the discharge chamber (anode), supported by an insulated and water-cooled electrode, there is a molybdenum cap of cylindrical shape R which covers the entire cross section of the discharge chamber. This electrode is at potential of the cathode; hence, primary electrons from the cathode, having passed through the ionization region, are "reflected" and returned to the discharge space of the chamber, executing multiple oscillations.

In order to avoid the possibility of ion charge exchange along the path from the region in which they are produced up to the exit slit of the source, the working cathode is positioned in such a way that the boundary of the primary-electron ionization region is directly against the plane wall of the discharge chamber in which the ion-exit slit is drilled; the thickness of the wall at the boundaries of the exit slit is made extremely thin for this purpose.

The use of an indirectly-heated cathode in the cyclotron source makes it possible to increase considerably the period of continuous operation as compared with a source in which a directly-heated cathode is used. Another, no less important feature of the indirectly-heated cathode is the fact that it allows the use of a simple electronic

scheme, in the cathode-heater circuit, for automatically maintaining a chosen mode of operation of the arc discharge from the instant the source is switched on until the working cathode disintegrates completely. This circuit is shown in Fig. 2.

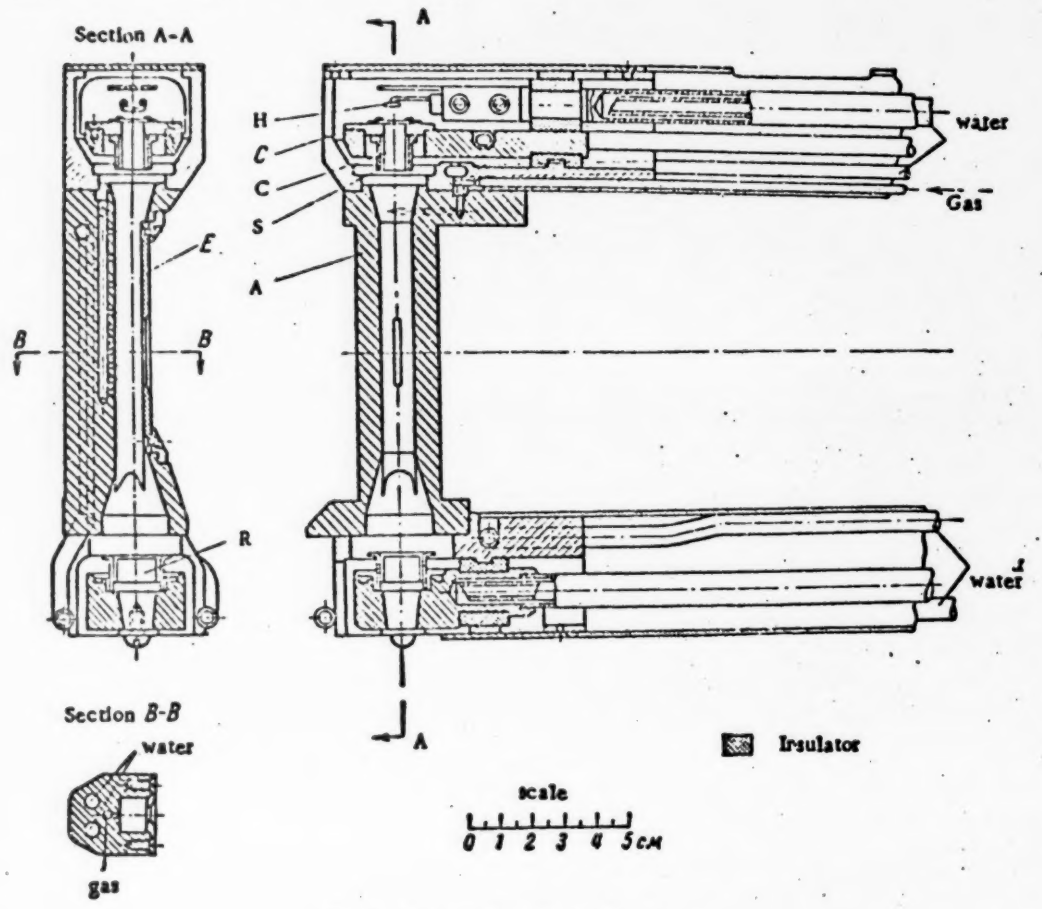

Fig. 1. Construction of the ion source.

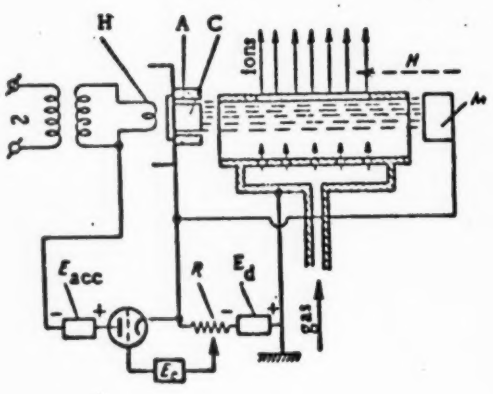

Fig. 2. Circuit of the source.

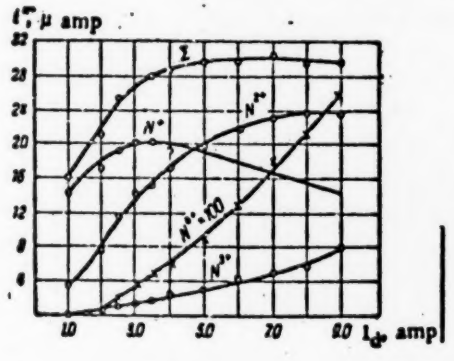

Fig. 3. Intensities of the currents of N^+, N^{2+}, N^{3+}, and N^{4+} ions and the total current Σ as a function of discharge current.

The ion spectrum produced in a prototype of this source was studied in a mass-spectrometer with 180° ion deflection in a uniform magnetic field. The field strength was 7,000 gauss (the source was located in the same field). The ions were accelerated by a potential difference of 30 kv.

The yield of N^+, N^{2+}, N^{3+}, and N^{4+} ions were plotted as a function of discharge current, discharge voltage and the flow of gas admitted to the chamber. Typical functions showing the absolute current strengths for N^+, N^{2+}, N^{3+}, and N^{4+} ions as a function of discharge current for a voltage of 300 volts are shown in Fig. 3.

The curves shown in this figure indicate clearly the growth of the current of multiply-charged ions as the discharge current is varied from 1 to 9 amp, and that this growth occurs more rapidly for higher ion charge multiplicities. Precisely this type of dependence on discharge current is to be expected on the basis of the step mechanism for the production of multiply-charged ions [2].

Along with the foregoing, it was also established that there are modes of operation of the discharge in which an increase of the discharge current causes a reduction in the output of multiply-charged ions. Curves illustrating this type of "anomalous" dependence of the spectrum on the discharge current are shown in Fig. 4, in which it is apparent that under these conditions there is a transition through a maximum simultaneously for N^{2+} and N^{3+} ions at values $I_d = 3.5$ amp and a sharp reduction in the intensity of these ions in the region of high discharge current.

This anomalous behavior is observed at small starting pressures in the discharge chamber when the density of ion current to the cathode is still comparatively small and when, in increasing the discharge current, one increases the external heating of the cathode in order to provide the required thermionic emission. (Non-self-sustaining discharge).

Measurements with a Lecher-line wave meter have shown that under these conditions radio-frequency oscillations in the decimeter range are generated and the intensity increases with discharge current. The production of radio-frequency oscillations was also found when there was a significant electron current to the reflector. In the mode in which intense oscillations were observed, the current to the collector was electronic current and the magnitude was comparable with the discharge current.

In going over to high pressures, the discharge gradually assumed the nature of a self-sustaining discharge exhibiting an increased ion current to the cathode; the ion heating effect at the cathode becomes comparable to that of the external source. In operation in the self-sustaining mode, the strong radio-frequency oscillations were not observed and the spectrum exhibited a greater abundance of multiply-charged ions.

The physical mechanism by which the yield of multiply-charged ions is reduced when strong oscillations take place was not specifically investigated. It may be assumed, however, that the oscillations lead to an increased electron diffusion rate across the magnetic field and thus tend to reduce the number of oscillations which the electrons execute in the direction of the field between the cathode and the reflectors.

Another basic discharge parameter which has a strong influence on the ion spectrum is the pressure in the discharge chamber. Curves showing the dependence of the spectrum on pressure are shown in Fig. 5 (I_d = = 5.0 amp; $V_d = 300$ volts). In taking these curves the pressure itself is not directly measured. The concept of a "pressure" has no direct meaning in this type of operation in which a high degree of ionization is achieved since the concentration of neutral particles is not the same over the cross section of the discharge chamber. Thus, a meaningful parameter in this case is the flow of gas q admitted to the chamber; q_0 is taken to be the gas flow for which the pressure in the discharge chamber is $\sim 5 \cdot 10^{-3}$ mm Hg.

It is apparent from the curves in Fig. 5, that as the gas flow is increased the current intensity for all ions changes monotonically; however, the nature of this change is different for the different ions. In the region in which the N^+ ions increase, the multiply-charged ions exhibit a reduced intensity which is stronger for ions with higher charge.

It appears that the chief reason for the deterioration in the spectrum of multiply-charged ions as the gas flow is increased, and consequently the reason for the reduction in the concentration of neutral particles, is charge exchange, that is, the process in which one or more electrons are captured by a multiply-charged ion in a collision with a neutral particle.

Measurements of the dependence of the spectrum on pressure, carried out over a wider interval of gas flows, indicate that for flows close to the minimum required for a stable discharge with given values of current and voltage, there is a sharp departure from the behavior shown in Fig. 5. If, in the beginning, the yield of multiply-charged ions increases in absolute value as the pressure is reduced, then at flows close to the minimum flow, there is a "break" at which the intensity of multiply-charged ions falls off rapidly.

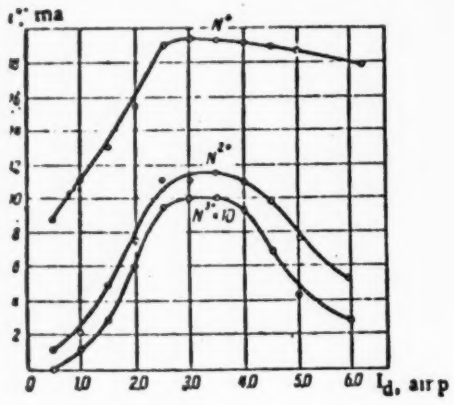

Fig. 4. Anomalous behavior of the dependence of the spectrum on discharge current.

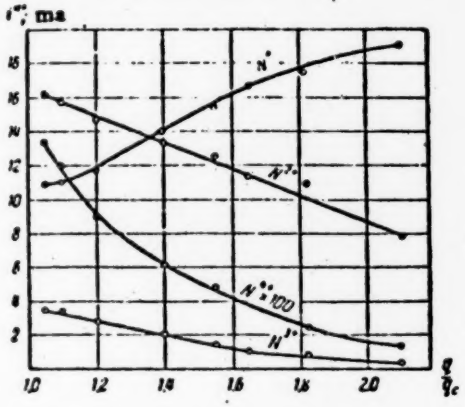

Fig. 5. Dependence of the spectrum on the flow of gas q admitted to the source.

The sharp reduction in intensity is accompanied by the excitation, in the discharge, of the intense oscillations which were mentioned above.

In trying to make measurements of the dependence of the ion spectrum on discharge voltage, it was found that the results could not be reproduced satisfactorily. Experimental data taken under the same conditions were found to differ, particularly in the most interesting region of discharge-voltage values (from V_d = = 300-600 volts).

In general, it is found that the dependence of the spectrum on discharge voltage is such that the intensity of multiply-charged ions tends to increase with voltage. The growth occurs more rapidly for ions with higher charge as is to be expected on the basis of the difference in ionization potentials for these ions.

Tests with a final model of the source, used subsequently in the cyclotron, were carried out with the same mass-spectrometer for the purpose of choosing the optimum discharge parameters corresponding to the greatest yield of N^{5+} ions.

To get a more reliable indication of N^{5+} ions the accelerating voltage was modulated at a frequency of 15 cps; this made it possible to pass the portion of the spectrum in the N^{5+} region past a narrow slit in the collector and thus to obtain on an oscilloscope screen a pattern of the spectral lines for N^{4+} and N^{5+} with completely satisfactory resolution.

The total current due to N^{5+} ions at the receiver was 100 μ amp and was, on the average, about 5% of the current due to N^{4+} ions.

In operations with the one-and-one-half meter cyclotron, the N^{5+} current at the internal target is, on the average, 1.5 μ amp at an ion energy of 105 Mev [3]. The ions are extracted from the source by a tantalum electrode which is placed in the dee, opposite the exit slit of the source.

When the discharge is operated at 4-5 amp and 750-800 volts the source can be operated continuously for 20-24 hours. The limitation is the rate of deterioration of the cathode, which under the operating conditions

indicated above is approximately 0.5 g/hr. If necessary the cathode operating period can be extended by using a cathode of larger dimensions. The average consumption of gas at maximum N^{5+} ion yield is 1 cm^3/min.

The cyclotron source was designed by Chief Engineer N. I. Chizhov.

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Sr^{90} YIELD IN U^{233} FISSION

M. P. Anikina and B. V. Ershler

The fragment yield in U^{233} fission has not been studied adequately. In particular, there is no data on Sr^{90} yield in the literature. However, the yield of this strontium isotope, which has a long half-life (19.9 years [1]) and a small cross section for activation by neutrons (1.0 barns [2]) is of great interest. The accumulation of Sr^{90} can be used to determine the number of fission events which occur in a sample of U^{233} under a prolonged irradiation, that is, under conditions when it would be difficult to measure the comparatively short-lived fragments, for example Ba^{140} (12 days) or Sr^{89} (53 days).

We have determined the Sr^{90} yield in prolonged irradiation of a sample of U^{233} as already described in [3]. As was reported, out of the entire sample of uranium, 60.75 mg, 11.9 mg of uranium underwent fission. The activity of the Sr^{90} in this sample was determined after a three-year exposure and the activity of Y^{90}, which was separated out from portions of the Sr^{90} was determined after a 20-30 day exposure, that is, after equilibrium was established for the Sr^{90} (19.9 years) $\rightarrow$ Y^{90} (65 hours). In addition, other samples of U^{233} were investigated which had been subjected to short-term exposures and the Sr^{90} yield was also determined.

Measurement No.	Yield calculated from Sr^{90} activity, %	Yield calculated from Y^{90} activity, %	Average yield, %
	I. Prolonged exposure of uranium sample		
1	4.61	4.45	4.56 ± 0.08
2	4.47	4.66	
3	4.57	4.56	
	II. Short-term exposure of uranium sample		
1		4.32	4.46 ± 0.14
2		4.38	
3		4.68	

In the latter experiments, the number of fission events occurring in the sample was determined from the amount of another strontium isotope, Sr^{89}, the yield of which, according to data given in the literature may be taken as 5.6% [4]. The results of the measurements are shown in the table.

Comparing the Sr^{90} yield for prolonged and short-term exposures, it is found that Sr^{90} and the nuclide with A = 90 which proceeds it in the chain have small cross sections for neutron absorption.

The most reliable value for the Sr^{90} yield which can be obtained on the basis of the data is (4.56 ± 0.08) percent found in the experiments with prolonged exposure of the uranium sample.

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RARE-EARTH ISOTOPE YIELDS IN THE FISSION OF Pu^{239} BY PILE NEUTRONS

L. M. Krizhansky, Ya. Maly, A. N. Murin, and B. K. Preobrazhensky

In 1956, several short preliminary reports on the mass-spectrographic determination of the yield of certain products of plutonium fission by pile neutrons were published. Fleming and Thode [1] determined the relative yield of the xenon isotopes [1], while Wiles, Petruska and Tomlinson [2] determined the yields of cesium, cerium, neodymium and samarium. The results of these measurements are shown by the solid line in the figure. The dotted line denotes the yield curve as obtained by radiochemical methods.

TABLE 1

Relative Yields of Isotopes of Cerium and Neodymium.

Isotope	Date of present experiment	Data of [2]
Ce^{140}/Ce^{142}	0.85 ±0.01	1.11 ±0.02
Ce^{142}/Ce^{142}	1.00	1.00
Ce^{144}/Ce^{142}	0.79 ±0.04[1]. [2])	0.77 ±0.02[1])
Nd^{143}/Nd^{143}	1.00 [3])	1.00 [3])
Nd^{144}/Nd^{143}	0.830±0.006 [1]. [2])	0.836±0.006 [1])
Nd^{145}/Nd^{143}	0.666±0.002	0.681±0.002
Nd^{146}/Nd^{143}	0.557±0.002	0.561±0.005
Nd^{148}/Nd^{143}	0.362±0.005 [4])	0.379±0.002
Nd^{150}/Nd^{143}	0.220±0.002	0.219±0.002

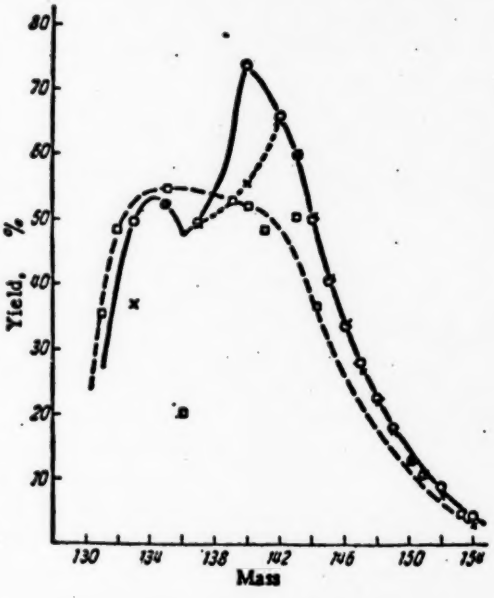
1) A correction is introduced for the decay of Ce^{144} (T = 282 days).

2) In calculating the correction for the Ce^{144} decay the burnup of Pu^{239} during irradiation was taken into account.

3) A correction is introduced for the reaction Nd^{143} (n, γ) Nd^{144} (σ = 334 barns).

4) A correction is introduced for contamination due to natural Sm^{148} (see footnote 4 in Table 2).

Without correction, the ratio Nd^{148}/Nd^{143} was 0.370 ± 0.005.



Yield curve in the fission of Pu^{239} by thermal neutrons in the mass region 130-154.
O – data of [2] (solid line); □ – radiochemical data (dashed curve); x – data of the present authors.

We have also undertaken a mass-spectrographic analysis of the yields of a number of products of Pu^{239} fission. As in [2], the plutonium was not separated from the fission products and the latter were subjected to

chemical fractionation but a direct mass-spectrographic analysis was made of the precipitate of a plutonium nitrate solution and its fission products, by changing (from element to element) the temperature of the source filament gradually.

In Tables 1-3 are shown the values of the relative yields of the various isotopes of cerium, neodymium, samarium and cesium. For purposes of comparison the data obtained by Wiles et al. are also shown.

TABLE 2
Relative Yields of Samarium Isotopes in Fission.

Mass	Data of present experiment		Data of [2]
	with a correction for contamination by natural samarium	without a correction for natural samarium	
147	1.52 ± 0.04 **, ***	1.54 ± 0.04 **, ***	1.56 ± 0.04**, ****
149	1.00	1.00	1.00
151	0.632 ± 0.01	0.602 ± 0.02	0.610 ± 0.02
152	0.414 ± 0.01	0.465 ± 0.02	0.492 ± 0.02
154	0.160 ± 0.003	0.220 ± 0.003	0.220 ± 0.009
148 *	–	0.034 ± 0.001	–

* Mass 148 due to contamination by natural samarium.

** A correction is introduced for the Pm^{147} decay (T = 2.52 years) [2].

*** In introducing the correction for the Pm^{147} decay the burn-up of Pu^{239} during irradiation was taken into account.

**** In [2] no correction was introduced for the natural samarium.

TABLE 3
Relative Yields of Cesium Isotopes in Fission

Mass	Data of present experiment	Data of [2]
137	1.00 *	1.00 *
135	1.06 ±0.10 **	1.058 ± 0.15 **
133	0.446 ± 0.002	1.005 ± 0.005

* A correction is introduced for the Cs^{137} decay (T = 26.6 years) [2].

** A correction is introduced for the reaction Xe^{135} (n, γ) Xe^{136} ($\sigma = 3.5 \cdot 10^6$ barns).

Aside from Ce^{140} and Cs^{133}, in all cases the results obtained in the present work and those obtained by Wiles are in good agreement.

The considerably lower value of the Cs^{133} yield obtained in the present work may be explained both by the contaminated natural caesium used in Wiles' work, and by the partial loss, in the present work, of the element which preceeds Cs^{133} - Xe^{133} (T = 5.2 days) during irradiation.

In the case of Ce^{140}, however, this last explanation does not apply since there are no long-lived gaseous or volatile elements among the preceding elements in the chain. It should be noted, that the total flux of

neutrons in the present experiments was considerably greater than in the work performed by Wiles, reaching $2.7 \cdot 10^{19}$ neut/cm^2 * with a target mass of 57.4 mg.

The cerium contents of the sample reached 0.5 mg and possible contamination by the natural cerium both in the target itself, and in the reactants could not be very large.

In the experiments performed by Wiles, the total flux was $2.2 \cdot 10^{19}$ neut/cm^2 and the target mass was 6 mg, that is, approximately, 9 times smaller; thus, the Ce^{140} yield in these experiments should be almost 100 times smaller and the natural contamination could possibly become noticeable.

The results obtained in the present work are shown in the figure by crosses. The points corresponding to the Nd^{143} and Cs^{137} yields coincide with the points of the corresponding yields obtained by Wiles.

The yields of the cerium isotopes have been normalized to the yields of Ce^{144} and Nd^{147} by the usual method [2]. The yields of the samarium isotopes (as was done in [2]) have been normalized so that they lie on a "smooth" yield curve.

The points corresponding to the yields of the xenon isotopes are not shown on the figure since their absolute value was determined by the Cs^{133} yield, the value of which in the present work, was very different from that obtained by Wiles.

A detailed discussion of the results from the point of view of the theory given by Pappas [6] should be deferred until additional experiments are performed. Nevertheless, one may presume the existence of a fine structure in the mass region 135-137 in fission of Pu^{239} by thermal neutrons.

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* This was measured by the reaction yield for Co^{59} (n, γ) Co^{60} (σ = 35 barns) [3]. The Co^{60} activity was determined by a 4π-counter and the γ activity by comparison with a standard radium sample. The accuracy of the determination was 1-2%. Virtually the same result was obtained by a direct determination of a flux using the isotopic composition of the plutonium after irradiation, as measured with a mass-spectrometer. The corrections for self-absorption of neutrons in the target (plutonium) were introduced in accordance with [4] and [5].

SHORT-TERM γ-RADIATION FROM FISSION PRODUCTS IN U^{235} AND Pu^{239}

O. I. Leipunsky, V. N. Sakharov, and V. I. Tereshchenko

The spectral composition of the short term γ-radiation of fission products is not very well known. This question was touched upon in [1] and [2] presented at the International Conference on the Peaceful Uses of Atomic Energy in Geneva in June, 1955. The investigation reported in the latter paper consisted of a study of the γ-radiation from a uranium target, irradiated in a reactor, which was analyzed using a luminescence spectrometer. It was found that there is γ-radiation with an energy of approximately 2.2 Mev, associated with a half-life of 0.5 sec; and also γ-radiation at a somewhat lower energy associated with a period of approximately 2 sec.

In this letter we report data obtained by the authors in 1953 pertaining to the spectral distribution of γ radiation from fission products of U^{235} in the time interval 1.5-5 sec immediately following fission. Certain results concerning the decay of the γ activity in the fission products of U^{235} and Pu^{239} as a function of time are also presented.

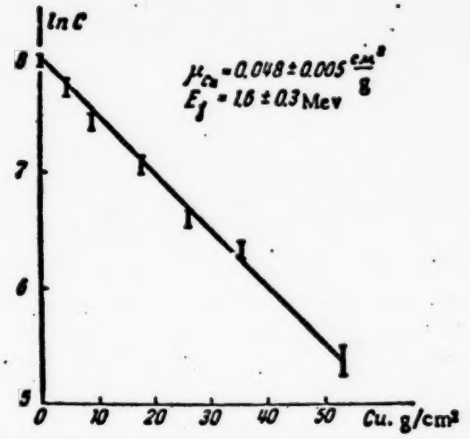

Fig. 1. Intensity of γ-radiation from fission products in U^{235} as a function of absorber thickness.

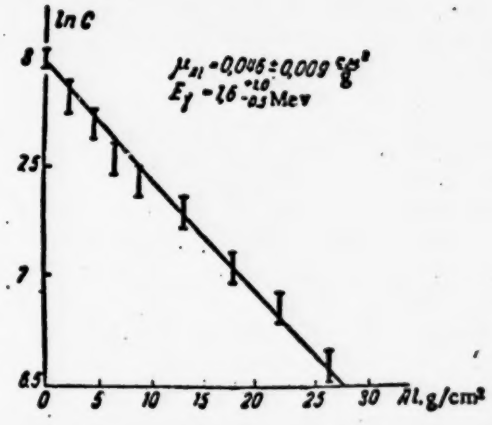

Fig. 2. Intensity of γ-radiation from fission products in U^{235} as a function of absorber thickness.

The "hardness" of the γ-rays was determined by absorbing a collimated beam in matter. The target of U^{235} or Pu^{239} was irradiated in the flux of thermal neutrons from the reactor by means of a pneumatic positioning device. The irradiation was carried out for a period of approximately 1 sec. The stability of the neutron flux during the time of irradiation was checked by a copper indicator (monitor) which was irradiated along with the target. After irradiation the target was transferred pneumatically to the measurement apparatus. The measurements were started approximately 0.6 sec following the end of irradiation. The measuring apparatus consisted of lead collimator, a copper or aluminum absorber and a steel counter SGS-400 in a glass vessel. It

was assumed, that the sensitivity of this counter to γ-rays was close to that of a copper counter [3], that is, it is independent of energy for γ-radiation with energies above 300-500 kev. The beta-radiation, originating in the irradiated target, was absorbed by the walls of the pneumatic instrument (3 mm of steel and 2.5 mm of aluminum). The counter pulses were recorded on film with a loop oscillograph. The time markers and the markers indicating the entrance of the target into the irradiation zone and its withdrawal were also recorded on this film. The count extended over a period of 5 sec, or in some cases, 17 sec. Thus, the quantities being directly measured were the countingrates as a function of time for absorbers of different thickness. Before the main measurements of these experiments, the apparatus was checked using sources of γ-rays with known energy, Co^{60} and Au^{198}.

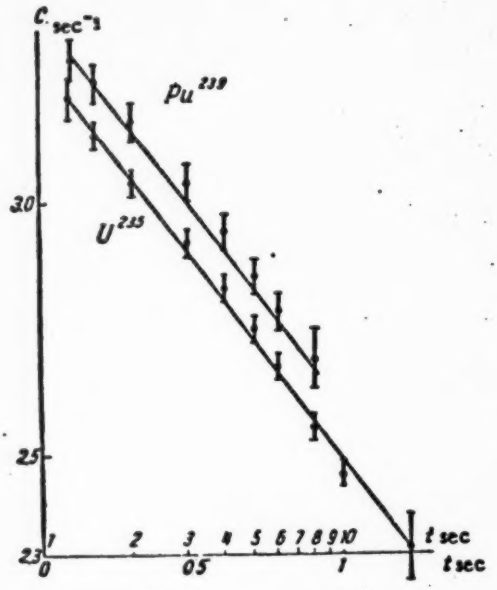

Fig. 3. Decay of γ-activity of fission fragments of U^{235} and Pu^{239} taking into account the correction for irradiation time.

From these measurements it is possible to obtain, first of all, the attenuation of the γ-ray beam as it passes through matter, and secondly, the decay of the γ-activity as a function of time. In Figs. 1 and 2 are shown the dependence of the intensity of γ-radiation of the fission products of U^{235} as a function of absorber thickness. The attenuation coefficient for γ-radiation can be determined from the slope of the line drawn through the experimental points. The mean energies corresponding to this coefficient [4] are indicated on the graphs. The principle result was obtained with a copper absorber, the thickness of which reached 50 g/cm^2.

The measurements with the aluminum absorber were carried out only up to 25 g/cm^2 and show a large spread in the determination of the average energy but serve as a check on the preceeding results. The errors in the value of the mean energy shown on the graphs should not be considered as the limits of the energy of the γ-radiation since the experimental data is not inconsistent with assuming that we are dealing here with a mixture of two or more types of γ-quanta (for example, 50% $E\gamma$ = 1.2 Mev and 50% $E\gamma$ = 2 Mev) although the mean energy lies within the limits indicated.

On the ordinate axis in Figs. 1 and 2 is plotted the logarithm of the number of pulses counted by the counter in a period of time from 1.5 to 5 sec after the middle of the irradiation period. A test showed that during this time interval the attenuation coefficient did not change as a function of time by more than the mean error, that is, the mean energy of the γ-rays is constant or varies to only a slight extent.

In Fig. 3 on a log-log plot is shown the decay of γ activity of fission fragments in U^{235} taking into account the corrections for irradiation time, that is, the decay of the γ activity after momentary irradiation. This relation is satisfactorily described in the time period from 1.25 to 17 sec following fission by a function of the form

$$t_{sec}^{-0.8} \pm 10\%.$$

In Fig. 3 are also shown the experimental data on the kinetics of the decay of γ activity of the fission fragments in Pu^{239}, which are also given by

$$t_{sec}^{-0.8} \pm 10\%.$$

that is, the kinetics of decay of the γ activity coincide in U^{235} and Pu^{239} within the limits of the experimental accuracy.

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MEASUREMENT OF THE CROSS SECTION FOR THE REACTION $Be^9(n, 2n)Be^8$ BY FISSION NEUTRONS

B. G. Dubovsky, A. V. Kamaev, and E. F. Makarov

Along with the other light elements, beryllium is used as a neutron moderator in nuclear reactors since it has good moderating properties and a small neutron-absorption cross section. Furthermore, two nuclear reactions occur in beryllium, $Be^9(n, 2n)Be^8$ and $Be^9(\gamma, n)Be^8$, as a result of which neutron multiplication occurs. However, the reaction $Be^9(n, \alpha)He^6$, also occurs in beryllium; this reaction tends to remove neutrons from the chain reaction process. Hence, the need arises for measuring the differential effect of these two processes.

The (n, 2n) reaction in beryllium was first observed by Rusinov [1]. All the subsequent measurements of the cross section for this reaction, with neutron sources characterized by different spectra, have given extremely inconsistent results [2-4].

An analysis of the possible reaction schemes and the disposition of levels in Be^8, Be^9, and Be^{10}, indicates that if neutrons with a broad-energy spectrum are incident on beryllium, then the spectrum of (n, 2n) neutrons will also be characterized by a broad-energy distribution [5-7]. The (n, 2n) neutron spectrum will approximate the incident spectrum. On the basis of these qualitative considerations it is valid to use a counter with an efficiency which depends weakly on neutron energy.

Method of measurement. The source of neutrons with a fission spectrum was a uranium converter, which was placed in the neutron beam issuing from a horizontal channel in the thermal column of the reactor at the atomic power station (Fig. 1). The converter was alternately surrounded by spheres of beryllium and graphite.

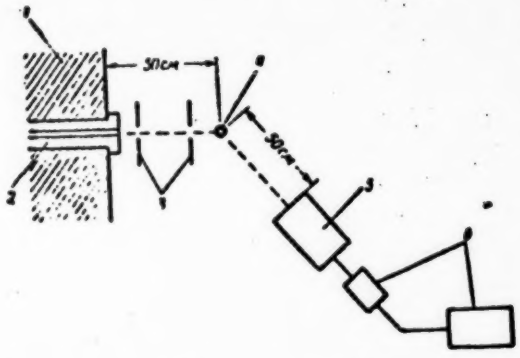

Fig. 1. Arrangement of the converter and counter.
1) Thermal column; 2) collimator; 3) cadmium diaphragms; 4) uranium converter at the center of beryllium sphere; 5) counter; 6) amplifier.

The neutron multiplication in beryllium due to the (n, 2n) reactions was measured by means of a neutron counter, the efficiency of which was weakly dependent on energy [8]. To check that the counter efficiency was essentially independent of neutron energy, a check was made of the constancy of the counter readings when the sources were enclosed by graphite spheres of various diameters. The maximum diameter of a graphite sphere was 200 mm. The uranium converter is shown in Fig. 2. The detection apparatus consisted of a neutron counter with BF_3 gas, an amplifier with a gain of $3 \cdot 10^3$, a standard scaler and stabilized power supplies. The stability of the apparatus was checked for several days using a Ra-α Be neutron source.

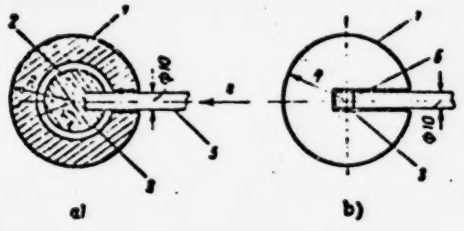

Fig. 2. Construction of the uranium converter and the beryllium spheres.
a) With the spherical converter; b) with a cylindrical converter; 1) beryllium sphere; 2) spherical uranium converter; 3) cadmium jacket; 4) thermal neutron flux; 5) aluminum tube; 6) cylindrical uranium converter.

Each run consisted of three measurements: a measurement with the beryllium sphere, a measurement with the graphite sphere and a measurement of the background with the beam of thermal neutrons cut off by cadmium. Each measurement lasted for 15 min. This sequence of measurements eliminated uncertainties due to any possible slow drift in the reactor power level (± 1%).

When the beryllium sphere was replaced by the graphite sphere, the background remained constant. In making the measurements the position of the spheres with respect to the converter was changed in order to avoid the effect of any possible asymmetry in the spheres. The measurements were carried out with 3 beryllium spheres with different wall thicknesses and 2 converters of different shape and size (Fig. 2). The spheres were fabricated from beryllium with a density of 1.77 g/cm^3.

To check that the fission-neutron source was isotropic, control experiments were carried out in which the angle between the counter axis and the direction of the thermal-neutron beam was varied.

Experimental estimates indicate that possible errors, due to a random shift of 2-3 mm of the effective center of the source with respect to the center of the sphere, are small as compared with the statistical errors. An approximate calculation showed that, in the direction of the thermal neutron beam, the fission-neutron flux is attenuated by a total of about 5% due to absorption in the converter. This effect should have no noticeable influence on the results of the measurements. To eliminate uranium fission by neutrons which have been slowed down in the spheres, the converters were covered with cadmium jackets 0.3 mm thick.

The experiments with the Po-α-Be and the Po-α-B neutron sources were performed in similar fashion. The sources were placed at the center of the spheres and the counter at different distances from the source (50, 100, and 150 cm). In all cases, the same value of the neutron-multiplication factor was obtained.

Results of the measurements. The neutron multiplication in the beryllium spheres is determined by the cross sections for (n, 2n), (γ, n), and (n, α) reactions. Since the number of γ-quanta with energies greater than the threshold for the (γ, n) reaction ($E_\gamma > 1.67$ Mev) produced in uranium fission is of the same order as the number of fission neutrons, while the cross section for the (γ, n) reaction is considerably smaller [7] than the cross section for the (n, 2n) reaction, the contribution of the (γ, n) reaction can be neglected.

The neutron counting level is related to the cross section for the (n, 2n) and (n, α) reactions by the relation

$$n = n_0 e^{Nl(\sigma_{n,2n} - \sigma_{n,\alpha})},$$

where n and n_0 are, respectively, the counting levels with the beryllium and the graphite spheres; N is the number of beryllium nuclei per cm^3; l is the mean path length for a neutron in a spherical layer of beryllium.

In view of the fact that the scattering of neutrons on beryllium nuclei tends to increase the mean neutron path, $\bar{l}$ exceeds the thickness of the beryllium layer by a wide margin $d = R - r$.

Neutron source	Dimensions of beryllium sphere		Number of counts (× 64) in 15 min.			$\bar{l}$ cm	ε %	$\sigma_{n,2n} - \sigma_{n,\alpha}$ 10^{-27} cm²	Remarks
	R cm	$d = R - r$ cm	beryllium sphere	graphite sphere	back-ground				
Spherical uranium converter	3.72	1.8	289	236	47	2.23	1.64 ± 0.3	61 ± 20	Average value over fission spectrum $(63 \pm 20) \times 10^{-27}$ cm²
Spherical uranium converter	4.25	2.2	144	142	26	2.87	2.05 ± 0.25	59 ± 20	
Cylindrical uranium converter	3.8	3.3	81	79	28	4.12	3.41 ± 0.44	68 ± 20	
Po-α-Be source	3.8	3.3	1470	1335	60	4.12	10.15 ± 0.75	204 ± 50	
Po-α-B source	3.72	1.8	227	229	54	2.23	−1 ± 0.5	−37 ± 25	

The mean path length $\bar{l}$ was calculated using the formula given by Yu. A. Romanov and V. I. Ritus:

$$\bar{l} = de^{-d/\lambda} + R \frac{\varphi(R/\lambda) - e^{-R/\lambda}}{(1 - e^{-R/\lambda})[1 + 1{,}5(r/R)^2]} \cdot (1 - e^{-d/\lambda}),$$

where

$$\varphi = 0{,}71 + 0{,}5^{R/\lambda} + 0{,}29e^{-0{,}45R/\lambda}.$$

The formula applies for $r/R \leq \frac{1}{2}$.

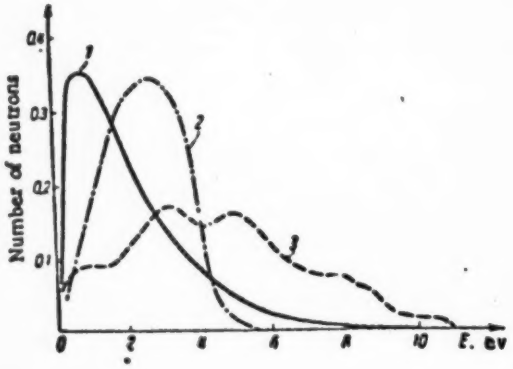

Fig. 3. Neutron-source spectra.
1) Spectrum for fission neutrons from U^{235};
2) spectrum from a Po-α-B source;
3) spectrum from a Po-α-Be source.

The transport mean-free path (λ = 7.3 cm) was determined from the mean scattering cross section in beryllium for neutrons with energies from 2 to 8 Mev taking into account the anisotropy in the scattering ($\sigma_s \approx 1.5 \pm 0.3$ barns). In the table are shown the experimental conditions, the average values of the quantity $\varepsilon = \frac{n - n_0}{n_0}$ and the calculated difference in the cross sections $\sigma_{n,2n}$ and $\sigma_{n,\alpha}$.

The error shown for ε is the mean-square error in the average value of ε taken over all the measurements. The uncertainty in the cross sections is determined by the statistical errors of the measurements and the uncertainty in the value of the scattering cross section in beryllium. Assuming $\sigma_{n,\alpha} = 10 \cdot 10^{-27}$ cm² for fission neutrons [9], we find $\sigma_{n,2n} = (73 \pm 20) \cdot 10^{-27}$ cm².

Comparison of these cross sections with the shape of the spectrum of the sources [10, 11] (Fig. 3) indicates that the main contribution to multiplication is that of fission neutrons with energies greater than 4 Mev and that qualitatively, there is a rapid growth in the difference of the cross sections for the (n, 2n) and (n, α) reactions as the neutron energy increases.

We were assisted in certain measurements by L. A. Geraseva and R. K. Goncharov.

In conclusion, the authors wish to express their gratitude to Doctor of Physico-Mathematical Sciences A. K. Krasin for setting up the problem and for a number of valuable comments and also to V. S. Stavinsky and E. Ya. Mikhlind for discussion of the results.

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K IONIZATION IN α-DECAY OF Po^{210}

V. V. Ovechkin and E. M. Tsenter

INTRODUCTION

According to Barber and Helm [1] the 75 kev x-rays which are emitted in the α-decay of Po^{210} arise as a result of internal conversion of 803 kev -rays and the ionization of the K-shell of lead.

The probability for internal ionization in α decay was calculated by Migdal [2] as early as 1941. However, a later calculation by Levinger [3] cast some doubt on the results given by Migdal, showing that the ionization of the K-shell should have a considerably smaller probability. An experimental test of the K-ionization effect in α-decay of Po^{210} is extremely difficult so that the γ-radiation conversion coefficient has not been determined very reliably.

For this reason the γ-radiation internal conversion coefficient in Po^{210} was remeasured by one of us and a qualitative estimate of the energy distribution of the ionized K-electrons was made [4]. The results obtained were in complete agreement with those of Migdal and are a convincing argument for the presence of a K-ionization effect in the α-decay of Po^{210}.

EXPERIMENTAL

a. Measurement of the conversion coefficient. The internal conversion coefficient $\alpha = \frac{N_e}{N_\gamma}$ for polonium γ-rays with an energy of 803 kev was measured with a scintillation spectrometer with a specially selected PEM-19 photomultiplier and a 6-channel pulse-height analyzer. A stilbene crystal, 3 mm thick, was used to detect the electrons; this size covered the entire range for conversion electrons ($E_e \sim 700$-800 kev). A sample of Po^{210} with a strength of 7.5 mc was precipitated from a polonium nitrate solution on a mica film 10 μ thick; the active surface on the side toward the photomultiplier was covered with an aluminum foil 10 μ thick.

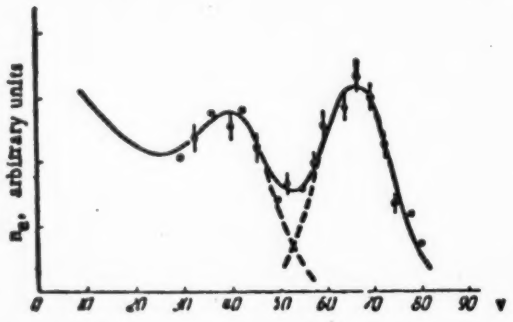

Fig. 1. Electrons spectra in Po^{210}. The pulse height in volts is plotted along the abscissa axis.

In Fig. 1 is shown the electron spectra obtained in Po^{210}. In the right hand part there is a clearly defined peak with half-width of 25%. Using control experiments (an energy analysis of electrons in coincidence with the x-rays) it was established that this peak is due to a conversion process; to the left there is the continuous spectrum of Compton electrons.

The number of converted electrons emitted per unit time by a polonium source, N_e, was determined by comparing the area under the peak with the area of the β-spectrum of a standard Co^{60} source, the intensity of which was measured previously using the count of (β - γ) coincidences. It was found that $N_e = (52 \pm 5)$ electrons per sec.

To determine the total number of γ-quanta (N_γ), emitted by the source the number of x-ray quanta N_x and the ratio $\frac{N_x}{N_\gamma}$ were measured. Measurements with new and old (2.5 years from the fabrication date) samples yielded the same value of $\frac{N_x}{N_\gamma} = 0.12 \pm 0.02$. This is in good agreement with the value 0.134 ± 0.025, obtained in [1]. N_x was measured for a good geometry in a spectrometer with a NaI(Tl) crystal and was found to be (420 ± 25) quanta per second.

The corresponding value of α is (1.5 ± 0.4) % . Using a direct measurement of N_γ and comparing the areas under the conversion curve and the Compton curve (Fig. 1) two more values of α were obtained: (1.8 ± ± 0.5) % and (1.2 ± 0.4) % . The average of these three measurements yields the value (α = 1.5 ± 0.2) %.

Because of the inadequate resolution of the spectrometer, the K and L electrons could not be separated in the spectrum (Fig. 1). Hence, to determine the conversion coefficient for the K-shell (α_K) the values of α we obtained were used in conjunction with the value of the ratio $(\alpha_K : \alpha_{L+M}) = 3.8 \pm 0.3$ from [5].

This procedure yields the value $\alpha_K = (1.2 \pm 0.3)\%$ which is in good agreement with the theoretical calculations by Rose et al. [6] which predict the value $\alpha_K = 0.95\%$ (E2 transition, $E_\gamma = 800$ kev).

b. The spectrum of electrons which are in coincidence with the K x-ray quanta. From the results of the present measurements of $\frac{N_x}{N_\gamma}$ and α_K it follows that only (10 ± 3) % of the x-ray quanta can be attributed to internal conversion of the γ-radiation. This, however, pertains to the residual part of the K x-rays; to verify that the x-rays were due to an ionization process, we measured the spectrum of the electrons which were in coincidence with the x-rays.

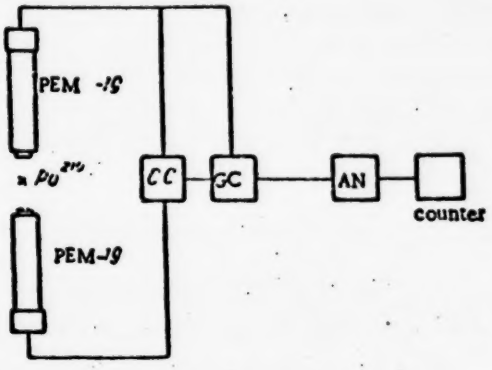

Fig. 2. Block diagram of the apparatus for measuring the electronic spectra by the (e − x)-coincidence method. C.C.) Coincidence circuit (τ = 0.15 μ sec); G.C.) gating circuit; AN 6 channel pulse height analyzer.

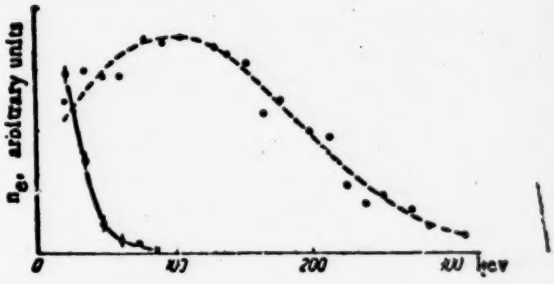

Fig. 3. Spectrum of electrons in Po^{210} in coincidence with the K x-rays in Po^{210}. The dots indicate (β − γ) coincidences in Co^{60}.

The upper counter with the stilbene crystal, 3 mm thick, was used to record electrons; the lower (with a sodium iodide crystal 3 mm thick) was used for the x-rays. The background due to α-particles was reduced to a considerable extent by means of the aluminum filter. The pulses from the upper multiplier were applied to the 6-channel analyzer through a gating circuit which was triggered by the pulses from the circuit which recorded coincidences between the electrons and x-rays.

The number of measured (e − x) coincidences as a function of electron energy is shown in Fig. 3. For purposes of comparison and calibration of the energy scale, in this same figure we show the β-spectrum from Co^{60} taken with an accuracy of approximately 7% by the (β − γ) coincidences in the same geometry. Using the Co^{60} spectrum and Curie plots, the electrons absorbed and scattered in the aluminum foil were determined.

The corrected spectrum for K-electrons from Po^{210} is shown in Fig. 4. In the energy region which was investigated (from 30 to 90 kev), the number of electrons in coincidence with x-ray quanta falls off monotonically with increasing electron energy; this is in qualitative agreement with energy distribution for ionized K-electrons calculated by Migdal (dashed curve in Fig. 4).

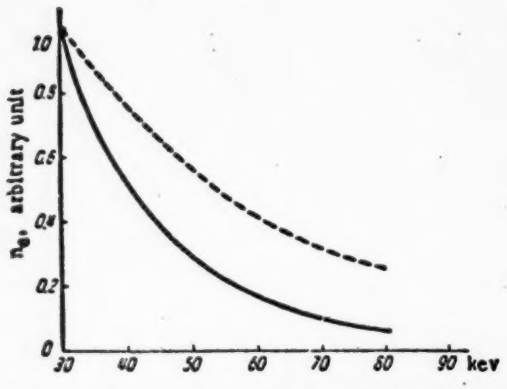

Fig. 4. K-electron spectrum. The dotted line is the data from Migdal [2].

Because of the poor resolution of the scintillation spectrometer and the necessity of introducing corrections for absorption, it is difficult to judge the extent to which the experimental curve agrees with the theoretical curve. Nevertheless, it is apparent that there is no significant disagreement between them. Thus, it may be asserted that the detected electrons are due to K-ionization and not to conversion. The following considerations also tend to support this conclusion.

The present measurements indicate that the intensity of (e − x) coincidences for electrons with energies from 40 to 90 kev is approximately equal to the intensity of (e − x) coincidences for an energy of approximately 720 kev. Since, in both cases, the coincidences between electrons and K x-rays were measured, this would indicate that the number of detected electrons with energies from 40 to 90 kev is equal to the number of K-conversion electrons. Actually, the number of "soft" electrons is somewhat higher since the aluminum foil passes only 62% of these electrons, and taking this effect into account the ratio of the intensity of "soft" electrons to the intensity of K-conversion electrons is 1.6: 1, with an error of 30-35%.

If the "soft" electrons are due to ionization, then, according to the theoretical spectra given by Migdal, in the region 40-90 kev these should constitute 22%, that is, the total number becomes 7.3 times larger than that of the K-conversion electrons. This value is close to the ratio of the intensities of x-ray quanta of "doubtful" and conversion origin (90: 10 = 9).

Inasmuch as each conversion K-electron of Pb^{206} corresponds to 1 x-ray quantum with an energy of 75 kev (the correction for the Auger effect in heavy atoms is approximately 11% [7]) this coincidence indicates that the x-ray intensity which is observed can be explained only if the low-energy electrons are due to ionization.

Comparison With Theory

A measurement of the spectrum of K-ionization electrons in the region from 60 to 240 kev has also been carried out by Lagasse and others [8]; the results are in good agreement with the Migdal theory.

In considering the measurement of the internal conversion coefficient α_K it is appropriate to carry out an estimate of the probability for K-ionization (P_K) in α-decay and to compare it with the value calculated by Migdal:

$$P_K = \frac{N_{ion}}{N_\alpha} = \frac{N_x - N_K}{N_\gamma} \cdot \frac{N_\gamma}{N_\alpha} = \left[\frac{N_x}{N_\gamma} - \alpha_K\right] \cdot k_\gamma. \qquad (1)$$

Substituting $(N_x : N_\gamma) = 0.134 \pm 0.025$ (from the data of Barber and Helm [1]), $\alpha_K = 0.012 \pm 0.003$ (from the author's data [4]), and $k_\gamma = \frac{N_\gamma}{N_\alpha} = (1.22 \pm 0.06) \cdot 10^{-4}$ (from the data of [9]), we find:

$$(P_K)_{exp} = (1.3 \pm 0.4) \cdot 10^{-4}.$$

Migdal [2], using an excitation potential of the form

$$V = 2[x^2 + y^2 + (z - vt)^2]^{-1/2} \text{ for } t > 0, \qquad (2)$$

obtained the expression

$$P_K = \left(\frac{v}{v_K}\right)^2 \frac{2.2}{Z^2} \tag{3}$$

where v is the velocity of the α-particle and v_K is the orbital velocity of the K-electron.

In Po^{210}, $v : v_K = 0.087$ and $(P_K)_{theo} = 2.4 \cdot 10^{-6}$, that is, it is approximately 1.5 times greater than the experimental value.

On the other hand, the results of the calculations by Levinger [3] are in sharp disagreement with the present data. In attempting to refine the calculation by Migdal, Levinger writes the Hamiltonian in the form:

$$H = 2[x^2 + y^2 + (z - vt)^2]^{-1/2} + \\ + (Z-2)[x^2 + y^2 + (z + v_n t)^2]^{-1/2}, \tag{4}$$

and obtains an expression, similar to (3) but containing in place of v, a factor $\frac{1}{2}[(Z-2)v_n - 2v] = \frac{1}{2} Zv_z$, where v_n is the velocity of the recoil nucleus, and v_z is the velocity of the movement of the center of charge. In Po^{210},

$$\frac{1}{2} Zv_z = v \frac{A-2Z}{A-4} = v(0.204). \tag{5}$$

Since the probability is proportional to v^2, replacing v by $\frac{1}{2} Zv_z$ in (3) reduces it by a factor of 25.

However, in this choice of the Hamiltonian account is not taken of the fact that displacement of the nucleus changes the position of the electron shell and that this change of position is related to the movement of the recoil nucleus.

The error in Levinger's work is easily explained by qualitative arguments. Consider an atom in a coordinate system moving with a velocity $-v_n$. In this system, an atom at rest has a velocity $+v_n$. The time origin should not be taken as the instant at which the α-particle moves through the potential barrier, but rather the time at which the α-particle reaches a velocity $\approx v$ (let us say 0.95 v), that is, the moment at which the α-particle goes through the steepest part of the Coulomb part of the potential barrier. Thus, at the instant $t = 0$, the α-particle is at a distance of the order of ten nuclear radii from the recoil nucleus. With this large distance between the α-particle and the recoil nucleus, the excitation influence of the electrostatic potential on the orbital electrons is negligible. At the instant of time $t = 0$, in the moving coordinate system, the recoil nucleus is at rest, that is, the effect of the decay process on the electron shell at the instant $t = 0$ will be the same as that which obtains when an atom moving with a velocity v_n is brought to a stop, which is many times smaller than the velocity of the K-electrons. Thus, processes occurring at the moment $t = 0$ cannot lead to any significant ionization. Starting at the instant $t = 0$, a calculation of the excitation can be carried out as was done by Migdal, with this one difference – in the moving coordinate system, where the recoil nucleus is at rest, the α-particle does not move with a velocity v, but with a velocity $v + v_n$, that is, a velocity $v\left(1 + \frac{v_n}{v}\right)$.

Since this correction is of order $\frac{4}{A}$, then taking into account the motion of the recoil nucleus leads only to a small change in the probability of ionization, which, however, does not lie outside the limits of the experimental accuracy.

Thus, the estimate of the probability for K-ionization in α-decay made by Migdal remains valid and is supported by the present experiments.

We wish to express our gratitude to Corresponding Member of the Academy of Sciences, USSR, A. B. Migdal for very valuable and stimulating discussions of the results of the present work and to Engineer V. V. Makarov-Zemlyansky for his suggestions concerning the construction of the apparatus.

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SCIENCE CHRONICLE

THE RIGA CONFERENCE ON THE USE OF RADIOISOTOPES

In December, 1956 a scientific conference, organized by the Latvian SSR Academy of Sciences, was held in Riga dealing with the use of radioactive isotopes in industry, biology and medicine. Workers of the scientific establishments and business concerns of Moscow, Leningrad, Tallin, Vilna and other cities also took part in the conference.

Opening the conference, Ya. V. Peive, president of the Latvian SSR Academy of Sciences and associate member of the USSR Academy of Sciences, pointed out that in the past few years the institutes of the Latvian SSSR Academy have made a series of investigations into the use of radioactive isotopes and radiations in research and in the national economy. The results of this research must be reviewed at this conference.

At the plenary session two papers were presented, "The Main Trends in the Application of Radioactive Isotopes in Automatic Control Equipment" by N. N. Shumilkovsky and A. Y. Meltser, and "The Use of Radioactive Isotopes in Medicine and Biology" by P. Y. Strazdun.

The conference was held in two sections, technical and biological. Twelve papers were presented at the technical sessions.

In his paper, "The Use of Gas Discharge Counters in a Radioactive Relay without Contacts," V. A. Yanushkovsky discussed the circuit principles of radioactive relays to register low and high intensities radiation, and also the question of using β-and γ-rays in work with radioactive relays. The author laid down operating conditions, referred to the methods of determining the necessary strength of the source to guarantee the reliable registration of discontinuous radiation and showed the possibility of increasing the resolution of gas discharge counters by using quenching, which limits the growth of the discharge in the counter, or by the system of using separate supply circuits for each counter. In addition, he reported on the application of a radioactive relay without contacts for the measurement and regulation of liquid levels, densities and temperatures.

V. V. Kruzhanski, I. I. Safyants, A. D. Tumulkan and V. A. Yanushkovsky devoted their paper to the practical use of radioactive methods for determining the quality of steel under production conditions at the Leningrad Molotov Steel-Rolling Mill. Instead of the earlier laboratory method for determining quality by the chemical analysis of metal samples, the radioactive method enabled the determinations to be made directly under production conditions in all the sections of the mill.

In their paper, "The Use of Radioactive Markers for the Construction of an Automatic Device for Changing from One Telecineprojector to Another in the Riga Telecenter," M. Bartushevich and I. A. Eymanis discussed the possibility of replacing magnetic by radioactive markers. The authors showed that the radioactive method is reliable in operation and does not require complicated equipment. It enables the conversion of the automatic operation of cineprojectors to a single system, both in the telecenters and in the general cinema circuit.

The paper of H. E. Gunne, A. D. Tumulkan and V. A. Yanushkovski, "Radioactive Indicators of Liquid Levels in Closed Vessels," was devoted to the possibility of constructing simple and portable equipment for the periodic measurement of levels. Among the circuits reviewed for devices, in which electrons are used and the indicator is a signal lamp, of practical interest is a system with a universal power supply and acoustic signals. The accuracy required in practice for level measurement is obtained by using a lead collimator in the apparatus. The paper, in addition, described the requirements for the least measurement error and the methods for evaluating the optimum values of the electrical and constructional parameters for devices of a given type.

B. E. Banashek and V. A. Yanushkovski in their paper, "A Radioactive Indicator of the Filling of Opaque Vessels in Continuous Production," gave an analysis of the possibilities for realizing a 100% continuous control of the filling of metallic or plastic vessels by the use of penetrating radiation and presented the principles of operation of an automatic tube-filling machine, which controls the filling of aluminum tubes in the cosmetic industry. This equipment sorts out the tubes by weight and makes an automatic count of the quality products.

The paper of Y. A. Auzan and P. F. Chaplinski dealt with the practical applications of radioactive regulating and signalling devices developed by the Physics Institute of the Latvian SSR Academy of Sciences in cooperation with the VEF works. Using a radioactive source, placed on a tilting arrangement of the measuring device, and a gas discharge counter, the positional regulation of the process controlled by the device can be achieved. Various circuits for radioactive relays using gas-discharge counters were discussed and their comparative characteristics were given. Examples of problems, solved using such devices, were given, and experience in their use showing their reliability was described.

V. O. Birshtein in his paper, "Experience with the γ-Ray Examination of Thin Metallic Sheets," discussed the advantages of using γ-rays as compared to x-rays for the examination of welds in metal tanks used for the liquefaction of fuel gas in a vessel-repair factory.

In the paper of H. E. Gunne, A. D. Tumulkan and V. A. Yanushkovsky "The Application of Scintillation Counters to the γ-Ray Flaw Examination," the various methods of constructing a portable device, which registers the average current from a photomultiplier, and the problem of obtaining the required accuracy when registering a defect of a given type were analyzed, and also, in the paper, the experimental characteristics of the scintillation counters were given, which allows the selection of the operating conditions of the photomultiplier.

The paper of L. L. Pelekis, Z. E. Pelekis and L. Y. Bambe, "The Determination of Steel Thickness by Scattered γ-Rays," was devoted to the study of the main advantages of this method. A scintillation γ-spectrometer was used for the detection of the scattered γ-radiation. The energy of the γ-quanta, scattered through the interval of angles 135-180°, is about 0.25 Mev and is practically constant. CsI(Tl) and NaI(Tl) crystals were used as scintillators.

Y. E. Chudars and I. Y. Taure in their paper, "The Attenuation of a Parallel Beam of β-Rays in Matter," gave an approximate method for the evaluation of the attenuation suffered by a parallel monoenergetic beam of β-rays with energies of up to 3 Mev. Two corrections introduced take into account the single collisions of the scattered electrons and their repeated deviation in the material. These calculations agree well with the experimental results of Bleiler, Tsyunti and Zemler.

V. Y. Gavrik, in his paper, gave examples of the construction of simplified nomogram for calculating the amount of radioactive isotopes.

In V. I. Breslav's paper, "Quantitative Analysis of Radioisotope Mixtures by Their Half-Lives," the method of analysis based on the differences in the decay rates was examined. This method, based on the well known procedure of a semi-logarithmic plot, gives good results when the half-lives of the isotopes in the mixture are different by 6-8%.

Exact values of the decay constants are obtained by the least squares method of fitting to the data. The measurements were made with a differential ionization chamber. The method is also suitable for use with Geiger-Muller counters.

At the meetings of the biological section 9 papers were presented, devoted to the results of research into the application of radioactive isotopes in the different branches of microbiology, chemistry, the physiology of plants and animals and medicine, which had been carried out in the Latvian SSR Academy of Sciences and the Riga Medical Institute. In some of the papers questions were discussed on the use of radiation to produce a definite action on living organisms.

Great interest was created by the communication from the workers of the Microbiological Institute (N. S. Stolugvo, I. V. Skard and others), who showed that, in principle, it is possible to obtain by the action of ultrasonics on a suspension of tuberculosis bacilli, absorbing radiophosphorus with their nutriment, labelled bacteria which would be free from radioactivity absorbed on their surfaces and which would contain all of the P^{32} in the

form of phosphorus compounds tightly bound to the cell material. The new method, developed by the above named scientists, for the preparation of labelled tuberculosis producing agents enables a successful solution of a number of problems, which have been sought for a long time, in the fields of the immunology and chemotherapy of of tuberculosis.

In the work described by V. J. Sergeeva and Z. I. Kreitsberg, the scientific associates of the Forestry Institute, the research workers were able with the help of radiocarbon-labelled pentoses and α-methylglucoside to trace the penetration of these substances into the lignin of the wood cell and thus clarify some of the previously unexplained peculiarities of the lignin structure.

In the same Institute, with the help of radiophosphorus, the dynamics of the transfer of chemical stimulators was established in pine trunks treated with these substances to increase the flow of gum (E. A. Rupays).

Interesting data on the calcium metabolism in hens was presented by the Biological Institute (V. K. Baumane). Radiocalcium made it possible to establish that the calcium, taken up by the hens in their food, is initially deposited in the skeleton, which acts as a store, and only from there goes to the egg shell and in this way is discharged from the organism.

A. G. Miller, an associate of the same institute, showed that a stimulation of growth in oats and other cultivated cereals is observed when the seeds are preliminarily soaked in an aqueous solution containing 0.1 to 1000 mc/liter of $Na_2HP^{32}O_4$.

F. G. Portnov, an associate of the Institute for Experimental Medicine, acquainted those present with an apparatus of his own construction to produce a measured ionization of respired air, the therapeutic action of which, especially in hypertonic disease, was proved by research carried out in the institute.

The other papers presented and the discussion which followed, indicated the current problems which faced the research workers in the further development of the investigations of the processes occurring in the living organism using labelled atoms.

In the resolution, accepted by the biological section, it was noted that in addition to the use of radioisotopes, it is also necessary to start work using stable isotopes and a mass spectrograph.

V. A. Yanushkovsky and S. A. Giller

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WITHIN THE SOVIET UNION

IN THE ATOMIC PAVILION OF THE ALL-UNION INDUSTRIAL EXHIBITION

SECTION "CHEMISTRY"

The successes of nuclear physics and the introduction of its results into the field of the peaceful uses of atomic energy are closely connected with the development of new and special trends in chemistry. The stands, presented at the exhibition, tell of the contribution of chemistry to the peaceful uses of atomic energy. Here,

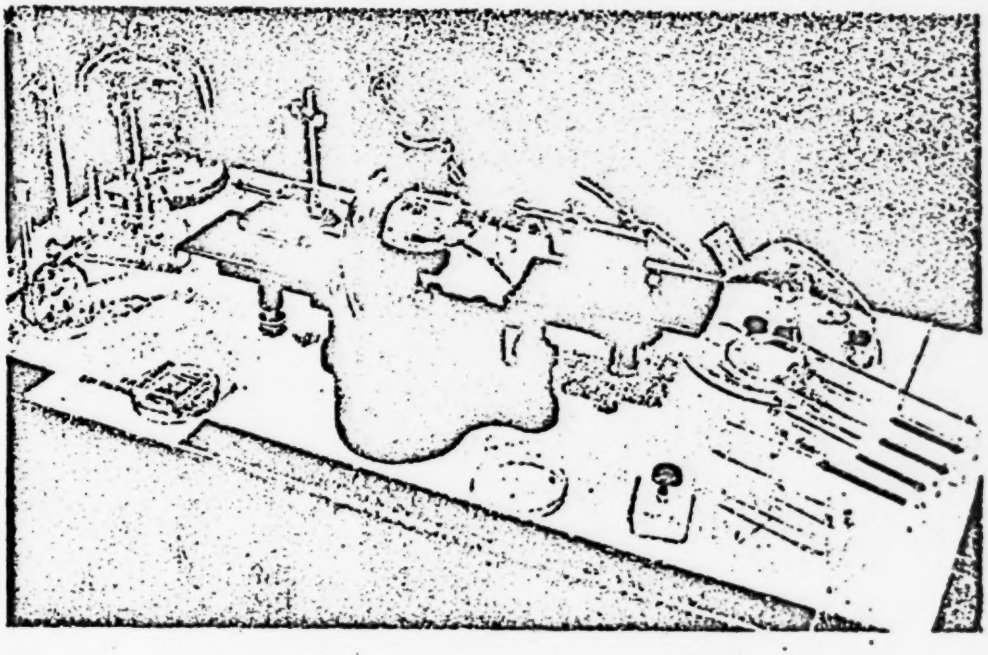


Fig. 1. General view of the laboratory for ultramicroanalysis.

there are models of the apparatus and equipment which make possible the extraction of radioactive and stable isotopes, the study of the action of radiation on matter, the analysis of the composition of naturally-occurring materials and their products, etc.

At the present time more than a thousand isotopes of the chemical elements are known. Their uses as labelled atoms bring further successes to physics, chemistry, biology and medicine. For example, the isotope B^{10} is used in neutron counters, C^{13}, N^{15} and O^{18} are widely used in the study of the life processes of plants and animals.

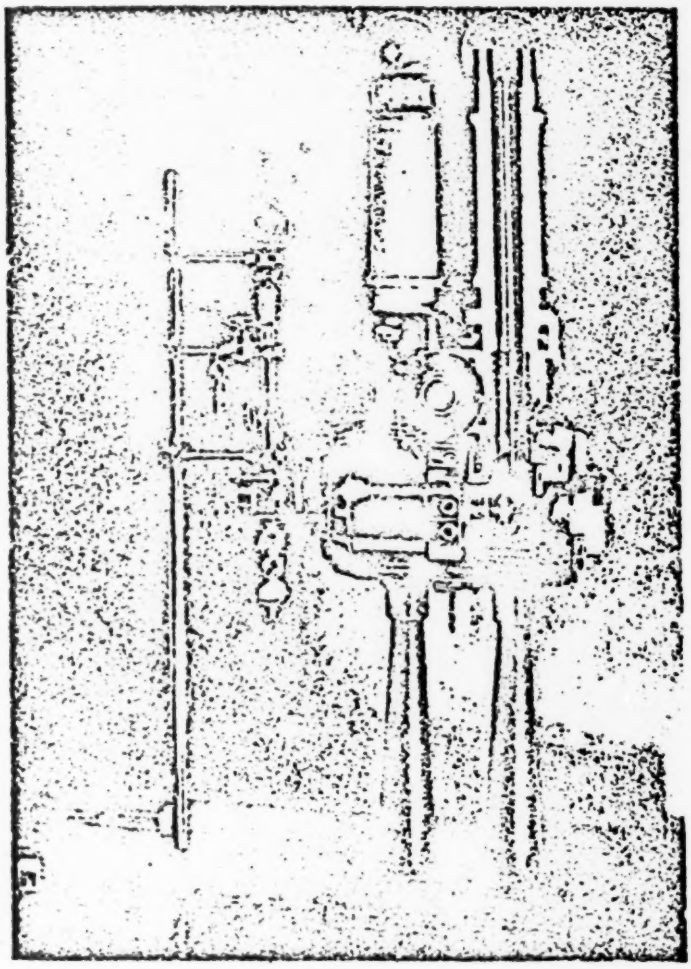


Fig. 2. Model of equipment with a Co^{60} source of strength 400-800 g-equiv. radium.

In modern practice different methods for isotope separation are used. A diffusion-separation column for the preparation of neon and argon isotopes can be seen at the exhibition. The column employs the counter-current method, in which a mixture of isotopes in diffusing through a stream of vapor is enriched with the light component. To obtain boron trifluoride, enriched with isotope B^{10}, a fractionating column is used. A working model gives a clear picture of the process occurring during the enrichment of the vapor. Models of equipment for the production of O^{18}, N^{15} and B^{10} are exhibited on the stand. No less interesting is the section illustrating the use of radioactive isotopes as labelled atoms in technology. A diagram showing the method of using

radioactive detectors during the different stages of the chromatographic separation of rare-earth elements is situated on the stands. A radiation counter enables the progress of the chromatographic experiment to be followed, by detecting the appearance in the measurement cell of an activity equal to, or greater than, an assigned value.

The apparatus for dealing with an ultra-small quantity of material is of great interest. Under a small transparent cover a type of microlaboratory is placed, in which all the materials are handled in the thinnest capillaries (Fig. 1). With such a laboratory it is possible to work with amounts of material weighing a millionth of a gram and to use volumes of the order of a thousandth of a milliliter (for example, in the investigations of the transuranic elements). At the same time, the concentration of a substance in solution becomes appreciable and the study of the various reactions can be made using the classical methods of inorganic chemistry. In the microanalytical laboratory all operations are carried out by means of micromanipulators on the microscope stage (which gives the impression of working in the scale of the usual experiments). The titration of solutions, electrolysis, pH measurements are made in vessels with a diameter of 0.5 1.0 mm. The burets for solution making are constructed according to the principles of capillary syringes. A centrifuge is used for the separation of precipitates from solution.

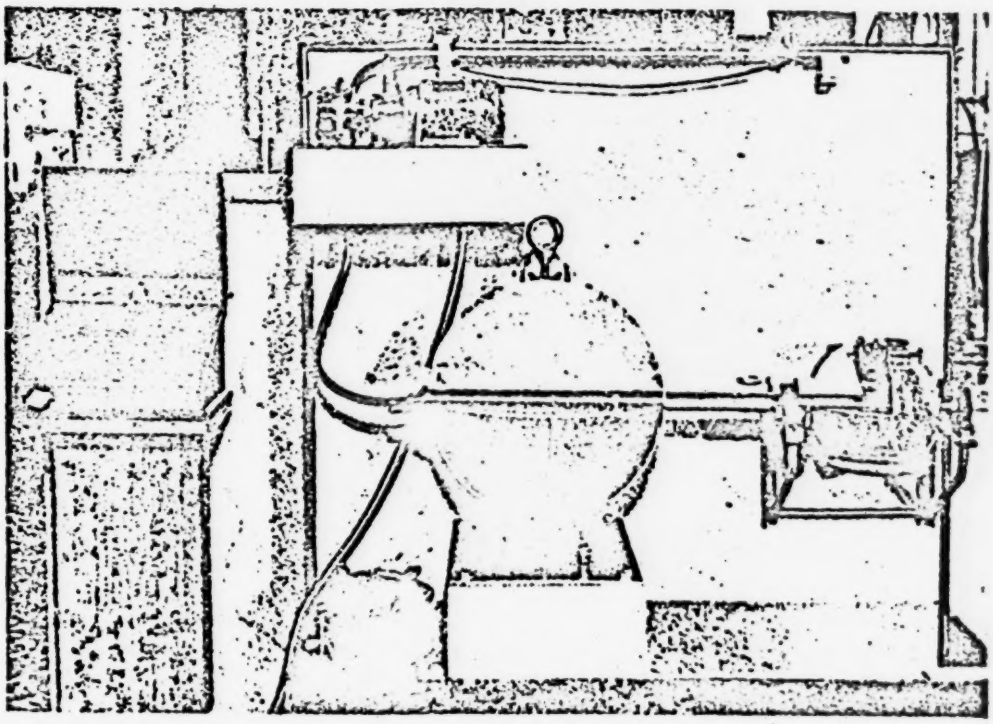


Fig. 3. Equipment for the determination of the quantity and composition of gases formed in the radiolysis of aqueous solutions.

One of the most sensitive methods of analytical chemistry – radioactivation analysis – is based on the property of isotopes to acquire radioactivity when irradiated by nuclear particles. If the sample to be analyzed is irradiated and then the radioisotopes formed identified by their radiation, it is possible to detect different elements even in the smallest quantities (from 10^{-4} to 10^{-11} g). The stages for conducting the radioactivation analysis are exhibited on the stand: a diagram of the reactor for the sample irradiation is shown, curves for the identification of the isotopes are given and the sensitivities of the different analysis methods are compared.

The necessity for studying the different chemical reactions that take place under the action of ionizing radiation has led to the appearance of a new branch of science – radiation chemistry. For experimental work in radiation chemistry Co γ-rays are widely used. Figure 2 shows a model of the Co source with an activity of 400–800 g-equiv. of radium. The equipment consists of two containers. One of them is used for the permanent storage of the radioactive material, the other (working) for the irradiation of samples. The samples are placed in the working container when it is at the top, outside the zone of radioactivity. The working container is then lowered into the receptacle into which the cobalt source is transferred by remote control from the storage container. The dose used is 100 r/sec. The equipment is supplied with block contacts which provide safety during work.

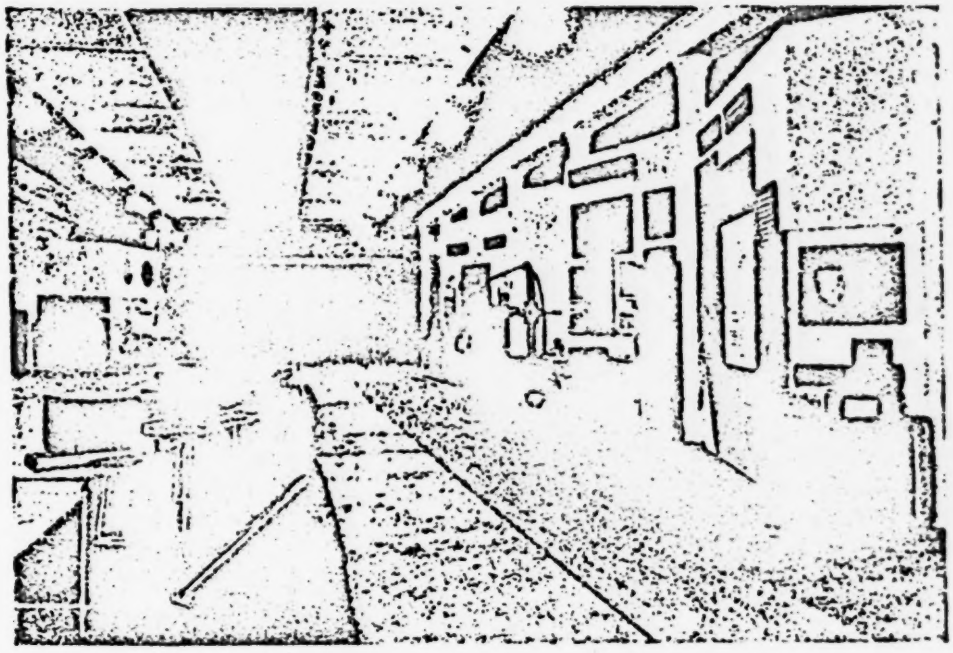
Other radiation sources are on display at the exhibition: x-ray tubes and also various devices used in radiation chemistry. One of these, designed for the determination of the quantity and the composition of gases produced by the irradiation of water and aqueous solutions is shown in Fig. 3.

The exhibition records the high level of development of the national science and technology in the field of the peaceful uses of atomic energy.

V. P.

THE LEIPZIG SOVIET SCIENTIFIC AND TECHNICAL EXHIBITION

"Atomic Energy for Peace" – thus was called the Soviet scientific and technical exhibition held in Leipzig (German Democratic Republic). The exhibits displayed there have demonstrated the considerable success of Soviet scientists in the field of the peaceful uses of atomic energy.



In the exhibition hall.

At the exhibition, officially opened on November 11, 1956 by F. Zelbman, the deputy chairman of the GDR Soviet of Ministers, the most varied fields of the application of atomic energy in the national economy were extensively presented. The visitor to the exhibition is acquainted with the general principles of the release of atomic energy and with a rich collection of uranium minerals. Visitors were particularly interested by the section describing the planned and operating nuclear reactors and atomic power stations, as well as the sections showing the wide application of radioisotopes in technology, industry and medicine. The exhibition has attracted the attention of German engineers, scientists and a large section of the community. The large number of visitors demonstrates the great interest in the achievements of Soviet atomic science: in 5 weeks the exhibition was attended by about 90,000 people, including many scientists from the GDR and FGR. The German press, as well as the exhibition visitors, highly rated the achievements of Soviet science.

For example, the newspaper "Neues Deutschland" of November 11, 1956 wrote:

"We have seen that the heritage of Pierre and Marie Curie, who discovered radium, of the English investigator Ernest Rutherford, who presented the theory of radioactive decay, and of many other prominent scientists, is safeguarded; nuclear physics has been raised to unprecedented heights by the Soviet scientists in the service of peace. The exhibition reflects an impressive picture of the peaceful application of atomic technology in the Soviet Union. The large number of varied instruments and devices, which reflects the diversity of the possible applications of atomic energy, and the objectivity of the exhibition, which excludes any implications of fantasy, are impressive."

"At the same time the exhibition testifies to the might of the atomic science of the peaceful USSR and testifies to the strength of the Soviet Union. The exhibition allows the scientists and the engineering-technical workers, as well as all workers to become thoroughly acquainted with the problems of the peaceful uses of atomic energy."

After seeing the exhibits, the visitor could view popular Soviet films which described the production of atomic energy and its wide applications.

The specialists who were interested in a particular section were able at any time to obtain information of interest to them from the Soviet specialists. Friendly meetings, mutual consultation, and the setting up of close contacts between Soviet and German specialists were continually the background to the work of the Soviet atomic exhibition in the German Democratic Republic.

B. A. Semenov.

THE FIRST CONFERENCE ON COSMIC-RAY PHYSICS IN THE GEORGIAN SSR*

At the end of November,1956, at the Institute of Physics of the Georgian SSR Academy of Sciences, in Tbilis, a conference on cosmic-ray physics was held, in which over 70 scientists from different cities of the Union took part.

The papers and discussions were mainly devoted to the three chief problems of cosmic-ray physics: the nature and the properties of the unstable elementary particles, such as mesons and hyperons, the nuclear interaction of cosmic rays and air showers.

Since the last cosmic-ray conference, held in Moscow in December 1954, this branch of science has considerably progressed and this was reflected in the papers presented at the conference.

The associates of the Lebedev Institute of the USSR Academy of Sciences reported on the results obtained with emulsion chambers exposed in the stratosphere; they were able to observe and analyze a number of interactions of primary protons, with an energy of 100,000 Mev, with nuclei of the photographic emulsion and on the basis of the analysis of these events were able to establish important new laws.

The associates of the Elbruz high-altitude laboratory presented an interesting paper in which the existence of new unstable particles was indicated.

The papers presented by the Lebedev Physical Institute, the University of Moscow, and the Physical Institute of the Georgian SSR Academy of Sciences gave important new data on the spatial distribution of particles in cosmic-ray showers.

New techniques in experimental physics were demonstrated at the conference.

The Physics Institute of the Georgian SSR Academy of Sciences and the United Nuclear-Research Institute reached an agreement on the joint work with the new gigantic accelerator being built in Dubno. A number of Georgian physicists will also work in an underground laboratory, built for the study of the most penetrating component of cosmic rays. In addition, a new high-altitude station at the Tskhra-Rskaro pass is being prepared for operation.

S. L.

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* E. Andronikashvili, "A long stage is passed," Dawn of the East 1956, December 1.

INSTRUMENTS FOR NUCLEAR PHYSICS

In the Department of Nuclear Physics of the Moscow Engineering-Physical Institute work has been carried out, during the course of a number of years, on the study of apparatus for the detection of ionizing radiation. The principles of diffusion chambers have been investigated and a number of chambers of different types have been investigated and a number of chambers of different types have been designed and built for physical research, the measurement of low activities and for demonstration purposes.

Because of the continuous mode of operation, the diffusion chamber as a demonstration apparatus has decided advantages as compared to the Wilson chamber. A chamber, with a transparent bottom, designed in the department, enables the observation of α-particle and electron tracks on a screen with an area of the order of 6 m^2. This chamber was demonstrated at the All-Union Industrial Exhibition, as well as the exhibitions dealing with the peaceful uses of atomic energy in Czechoslovakia, Sweden, Yugoslavia, Bulgaria and the German Democratic Republic.

A new type of diffusion chamber allows activities of the order of 10^{-12} curies to be measured with a high statistical accuracy because of the prevention of condensation on the surface of the object introduced into the chamber. One of the chambers for the measurement of low activities will be exhibited at the All-Union Industrial Exhibition, as well as at the international exhibitions on the uses of atomic energy for peaceful purposes.

A series of questions arise in the design of large diffusion chambers. The construction of a diffusion chamber with an area of 600 x 900 m^2 has been worked out in the department, the continuously-operating vapor source used being in the form of moisture-conducting plates placed at the two opposite walls of the chamber. This chamber is displayed at the All-Union Industrial Exhibition.

An experimental amplifier, with drift correction by means of an amplifier with a contact converter, has been developed in the department. It has been established that the sensitivity of the device is limited by the production of a charge on the converter contacts, when they are opened, of about 0.6×10^{-13} coul at a working frequency of 5 cps. The use of gold contacts lowers the charge and current by a factor of 5. Investigations have shown that the use of a contact converter is only worth while when measuring currents not lower than 10^{-11} amp per full-scale deflection.

V. Lyapidevsky and K. Ergis

THE DETERMINATION OF THE BORON CONCENTRATION OF MINERAL SAMPLES BY A NEUTRON METHOD

At the V. I. Vernadski Institute of Geochemistry and Analytical Chemistry, in the laboratory headed by Professor V. I. Baranov, scientific associates V. K. Khristianov, G. I. Panov, and A. A. Chernov are developing a method for the quantitative analysis of boron in mineral samples by means of neutron absorption.

The equipment used for this purpose consists of a neutron moderator and a detector. A Po–Be source of activity 2 curies is used as a source of neutrons. The neutron detector used was a small ionization chamber filled with B^{10} enriched BF_3. The current from the chamber served as a measure of the ionization produced by the reaction $B^{10}(n, \alpha) Li^7$.

The method has been tested in a geological survey, using 2700 samples, the accuracy of the method being maintained at 10%, by comparing it with a chemical analysis of 150 samples with a boron concentration of up to 14% B_2O_3, and the results showed that: 1) the neutron and the chemical analyses agree to within 20%; and 2) the error in reproducing results, for anhydrite concentrations above 5%, is not higher than 10%.

V. B.

THE USE OF RADIOACTIVE ISOTOPES AND RADIATION FOR OIL PROSPECTING IN THE USSR

Radioactive techniques have received a wide development and industrial application in the oil-producing industry of the Soviet Union. For radioactive bore-hole testing the methods used are based on the measurement of the natural, secondary and scattered γ-radiation, as well as neutron methods.

In oil geology, γ-ray bore-hole testing is combined with neutron bore-hole testing.

Neutron-γ bore-hole testing (NGT) was first proposed in 1941 by the famous physicist Bruno Pontecorvo. In the Soviet Union the work of developing the NGT method and equipment was done by a number of groups – NIIGR, CNIL of the combine "Azerbaijan Oil Geophysics" and others. The most important results were obtained in the laboratory headed by associate member of the USSR Academy of Sciences, G. N. Flerov and Professor B. B. Lapuk, where the NGT method and the equipment, useful for a wide industrial application, have been developed.

In NGT, the intensity of secondary γ-radiation, produced by the capture of thermal neutrons by the nuclei of the rock, is measured.

The differentiation by means of NGT is determined by the neutron slowing down and absorption properties of the rock, as well as by the different radiation properties of the intermediate nuclei formed by neutron capture.

Hydrogen slows down neutrons anomalously. This fact enables, in principle, the use of NGT to determine the hydrogen content of strata and to identify porous layers filled with oil and water, as well as layers of clay saturated with water and those containing water of crystallization.

The measurement of the natural and induced γ-radiation in bore holes is made simultaneously with the help of a double-channel apparatus.

NGT – combined with GT – enables a study to be made of the geological cross section of the bore hole, which has been drilled without extracting the core, even when the bore hole has been reinforced with a steel tube, it enables the separation of dense from porous strata and the separation of gas-bearing from oil-bearing strata. The neutron-γ method is widely used in practically all of the oil-bearing regions of the Union.

In the last few years in the Oil Institute of the USSR Academy of Sciences new equipment has been developed for NGT using scintillation counters, which has greatly increased the potentiality of this method. This equipment is being successfully used in production. This laboratory also carried out work on the development of the NGT method, combined with γ-ray spectroscopy (NGT-S), for the determination of the position of oil-water interface.

Gamma-gamma bore-hole testing (GGT) is based on the measurement of scattered γ-radiation which is produced by the interaction with the rock of the γ-rays emitted by the source lowered into the bore hole.

The intensity of the scattered γ-rays diminishes as the density of the irradiated rock increases.

GGT is beginning to be used in the oil industry for the study of the porosity of the oil-bearing strata. The theory of the method for solving the problem indicated is being developed by the engineering physicists of the combine "Bashkir Oil Geophysics" and the VNIIG* geophysicists Ya. A. Gulin, N. Y. Dyadkin and E. N. Filippov.

The method of neutron-neutron bore-hole testing (NNT) is based on the measurement of the density of slow neutrons in the substance surrounding the detector. The development of this method is being made in the USSR

* 'VNIIG' means 'All-Union Scientific Research Institute for Hallurgy.'

Academy of Sciences Oil Institute, where the equipment has been created employing neutron proportional counters as detectors. The equipment has been tested industrially with success and a test batch will be released by the factory "Oil Equipment." The Tatar, Bashkir, Kuybushev and Krasnodar geophysical combines are taking an active part in these investigations.

The investigations which have been completed enable the NNT method to be effectively used for the separation of water and oil-bearing strata in bore holes. The specific absorption of thermal neutrons in the water-bearing part of a layer is considerably bigger than in the oil-bearing part because of the presence in the water of dissolved salts (mainly NaCl).

The possibility of using thermal neutrons, and particularly neutrons of higher energy, in conjunction with NNT for the quantitative measurement of the porosity of oil-bearing strata, is also being investigated.

The method of induced activity (IA) is being developed in the laboratory for radiometric methods of the USSR Academy of Sciences Oil Institute, under the supervision of the associate member of the USSR Academy of Sciences, G. N. Flerov and Doctor of Geo-mineralogical Sciences, F. A. Alekseev. This method is necessary for the location of oil and water-bearing strata in bore holes reinforced with steel tubes, for the location of water-oil interfaces in experimental and production oil wells, and for the analysis of minerals.

The methods which have been developed for the separation of oil and water-bearing strata using activated Na^{23} and Mn^{55} are being widely used industrially in the Bashkir and Tatar oil-bearing regions.

Engineer physicist E. B. Blankov is successfully developing a faster method of induced activity, using Mn^{55} and Al^{27}, for the determination of the shifting of the water-oil interface suitable for use in the conditions of the Romashkin oil-bearing region.

The Oil Institute of the USSR Academy of Sciences and the combine "Krasnodar Oil Geophysics" are studying the use of the method using Na^{23} induced activity for the location of water-oil interfaces in oil-bearing regions in which the water is only slightly mineralized. Similar work is being planned for the West Turkmen oil-bearing regions in 1957.

The USSR Academy of Sciences Oil Institute is carrying out investigations into the use of the method of induced activity for the quantitative measurement of the oil content of strata.

An oil-well neutron generator is being developed by the USSR Academy of Sciences Oil Institute together with the VNIIG of the Ministry of Oil Production of the USSR and a number of specialized institutes.

At the present time, work on the application of radioactive isotopes is being carried out in almost all of the USSR oil-bearing regions.

The method is being developed in the VNIIG of the Ministry for Oil Production of the USSR.

With the help of radioactive isotopes the following problems are being successfully investigated:

1. The determination of the cement layer height in the space behind the pipe and a qualitative evaluation of the well cementing; the determination of the quality of cementing after repairs; 2) the location of the position of water seepage and the detection of the presence of circulation outside the pipe; 3) the location of the position where the bore solution is being absorbed in the case when circulation is lost during oil-well drilling; 4) the examination of the condition of pressure wells and the location of positions where the water being pumped in is being absorbed; 5) the detection of zones of hydraulic rupture of strata and the collection of quantitative data about its nature.

The I. M. Gubkin Moscow Oil Institute is carrying out experimental work on the use of radioisotopes for the evaluation of the permeability of strata during the process of drilling or before the lowering of the column.

The Oil Institute of the USSR Academy of Sciences has developed the method and equipment for the use of tritium for the observation of the movement of subterranean water.

In one of the laboratories under the supervision of the author and G. N. Flerov, associate member of the USSR Academy of Sciences, during the last few years the method of surface and aerial radiometric prospecting for oil is being successfully developed.

F. A. Alekseev

THE STUDY OF PROPANE CRACKING WITH THE HELP OF RADIOCARBON

In the USSR Academy of Sciences Institute of Chemical Physics work is in progress to investigate the mechanism of propane cracking.

Labelled ethylene was used to solve the problem of the way in which ethane is formed. By adding a small quantity (1%) of $C^{14}H_2=C^{14}H_2$ to the propane and analyzing the cracking products, N. I. Medvedeva and E. S. Torsueva found that the ethane shows a considerable activity.

To obtain quantitative data the authors used Neumann's kinetic labelled-atom method. From the appearance of the specific activity curves for ethane and ethylene formed by propane cracking in the temperature interval 530-580° C, they found that, within the limits of experimental error, ethane is produced only from ethylene. The same conclusion was obtained from a comparison of the rates of consumption of C_2H_4, calculated using the kinetic method equations.

In contrast to Rays, who explained the formation of ethane by the recombination of methyl radicals, Medvedeva and Torsueva propose the following scheme:

$$\begin{aligned} C_2H_4 + H &\rightarrow C_2H_5 \quad (E < 5\,\text{kcal}), \\ \dot{C}_2H_5 + C_3H_8 &\rightarrow C_2H_6 + \dot{C}_3H_7 \end{aligned} \tag{1}$$

or

$$C_2H_5 + H_2 \rightarrow C_2H_6 + H. \tag{2}$$

The authors were able to evaluate the concentration of hydrogen atoms and propyl radicals in the system at 580° C.

It is known that the propyl radical decomposes into methyl and ethylene according to the scheme

$$C_3H_7 \rightarrow CH_3 + CH_2{=}CH_2 \tag{3}$$

with an activation energy of about 28 kcal.

The rate of formation of C_2H_4, according to Reaction (3), is

$$\frac{d[C_2H_4]}{dt} = K[\dot{C}_3H_7],$$

where $[C_3H_7]$ is the required concentration of C_3H_7; $\frac{d[C_2H_4]}{dt}$ is the rate of formation of ethylene calculated from the kinetic method equations; and K is the monomolecular reaction rate constant.

The concentration of $\dot{C}_3H_7$, found in this way, is equal to 10^{10} radicals/cm^3.

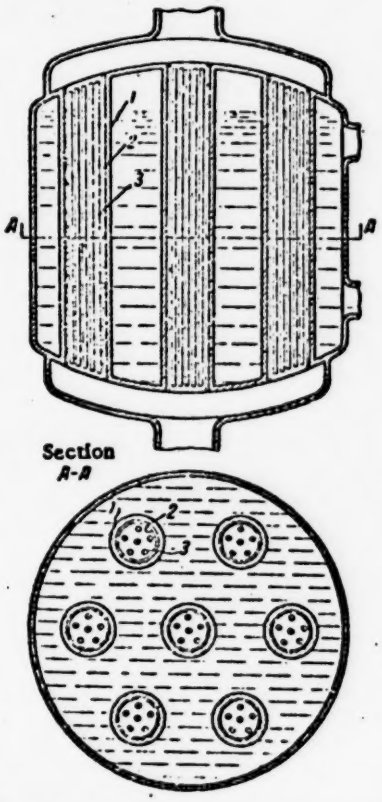
The hydrogen-atom concentration, determined from Reaction (1), was found equal to 10^{10} atoms/cm^3 with an activation energy of 5 kcal and the steric factor of 10^{-2}.

N. Serdyuk.

FOREIGN SCIENTIFIC AND TECHNICAL NEWS

A REACTOR WITH SODIUM AS HEAT-TRANSFER MEDIUM AND HEAVY WATER AS MODERATOR

The Atomic Energy Commission of the U.S.A. has approved a proposal for the planning and construction of a nuclear power reactor with heavy water as moderator, sodium as heat-transfer medium, and slightly enriched uranium as fuel [6].



Schematic representation of the reactor with sodium heat transfer medium and heavy water moderator.
1) Thermal insulation; 2) stainless steel tube; 3) fuel elements.

It is stated that the idea for such a reactor is not new; it was first proposed as early as 1940 by a group of scientists at Columbia University, for the motor of a submarine. But at that time the difficulties of solving a number of serious technical problems relating to the construction of such a reactor, and also the high priority given to the work on development of the atomic bomb, pushed the development of a reactor of this type into the background [1]. In 1954 the Swedish corporation "Allmanna Svenska Elektriska" (Vastiras) took out a patent on a reactor of this type [2].

The proposal to construct a reactor with sodium heat transfer medium and heavy water moderator was presented jointly by the concerns "Chugach Electric Association" and "Nuclear Development Corporation of America" [3]. It has been decided to build the reactor in Anchorage, Alaska, and it is to be put in operation in 1962. The electric power of the reactor is to be 10 megawatts. The exhaust steam can be used directly for home and public heating.

Besides other advantages, this reactor is very suitable for operation in remote regions where the cost of other forms of energy is very high (for example, in Alaska); the fuel used can be uranium with enrichment to less than 1.5%; and the reactor can thus be an article of export to such countries [3].

The renewal of the charge of fuel in the reactor will be carried out once every two years [1], so that the burning out of the U^{235} will come to 50%, and that of the Pu^{240} to 25%, with a coefficient of plutonium breeding of 0.75 [3]. It is assumed that the electric power obtained from this reactor will be cheaper than power developed by hydroelectric stations located in the same regions, which at the present time costs 1 to 1.2 cents per kwh.

With the further perfection of reactors of this type the cost of electric energy can be brought down to 0.7 cents per kwh, even for reactors of electric power less than 10 megawatts.

Despite the fact that reactors with sodium heat-transfer medium and heavy water moderator can operate on slightly enriched or natural uranium, which is very important for countries that do not have large facilities for separation of isotopes and production of fuel, they have the disadvantage that their use depends to a considerable extent on the availability of heavy water. For example, the cost of the heavy water used by the U.S. Atomic Energy Commission comes to 28 dollars per pound.

The work on the development of such a reactor will be carried out in three stages: first, research and schematic planning; second, research and technical planning; third, construction starting, and operation. After the completion of each stage there will be a study of the results of the work and also the question of the feasibility of continuing it. For carrying out the first stage of the work, 2,225,000 dollars are being sought from the Atomic Energy Commission. The estimates for the later stages of the work (research, construction of the reactor, and also work on perfecting it after the completion of the research and construction) come to 18,325,000 dollars. According to the agreement with the Commission, the firm "Chugach Electric Association" will provide the site for the reactor, the turbogenerator with its auxiliary equipment, and the construction of the necessary buildings. For these purposes the company proposes to spend 1,830,000 dollars [1].

The main advantage of the reactor in question is that it combines the use of the best of all moderators – heavy water – and the best of all heat-transfer media – sodium. But despite the fact that each of these materials taken separately has been well studied, their use together in a reactor requires the carrying out of a considerable amount of research and development. The most serious problem is that of avoiding contact between the sodium and the water, which would bring about a violent chemical reaction. In water-sodium heat exchangers this danger can be reduced to a minimum by suitable precautionary measures. But the deformation of the fuel elements brought about by the formation of fission products in the active zone of the reactor, and also other causes, can lead to the breaking of the channels which carry the fuel, and along which the sodium flows, and to serious consequences. Nevertheless, methods exist for overcoming this difficulty also [1].

The reactor takes the form of a vessel of diameter 240-320 cm. The heat developed in the moderator is removed from the reactor by the heavy water, which circulates through a heat exchanger. The fuel elements, 1.8 cm in diameter, covered with stainless steel and collected into bundles, are carried in vertical channels of stainless steel, 240 cm long.

The sodium entering the reactor goes into the lower collector, passes through the tubes, and comes together again in the upper collector (diagram). The channels carrying the fuel are carefully isolated from the surrounding water. Each tube is also enclosed in thermal insulation. The tank of heavy water is surrounded by a graphite reflector, which in turn is surrounded by a thermal neutron shield of iron 24 cm thick. The biological shield is made of heavy concrete.

In its specified dimensions this reactor is even somewhat smaller than reactors of other types, in spite of the fact that they operate on highly enriched nuclear fuel.

The reactor also has other advantages: 1) the ability to operate on slightly enriched uranium, and, at electric power not below 50 megawatts, even on natural uranium; 2) the choice of power for the reactor does not involve essential changes in its construction; 3) the service span of the reactor is fairly long, and the enrichment of the fuel is insignificant; 4) there is no need to reprocess the irradiated fuel, since at the low U^{235} content and the high percentage of burning out the reprocessing of the fuel is not economic; 5) the possibility of attaining very high temperatures of the heat-transfer medium, which permit the use of up-to-date turbines; 6) the lack of need to hold a high pressure in the reactor vessel, which lowers the cost of construction of the case and makes the servicing of the whole installation easier; 7) the use of heavy water as moderator makes the reactor self-regulating because of the negative temperature coefficient of reactivity. Moreover, the regulation can be accomplished also by varying the level of the moderator.

Yu. K.

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[2] Engineering 182, 4723, 346 (1956).

[3] Nuclear Eng. 1, 7, 311 (1956).

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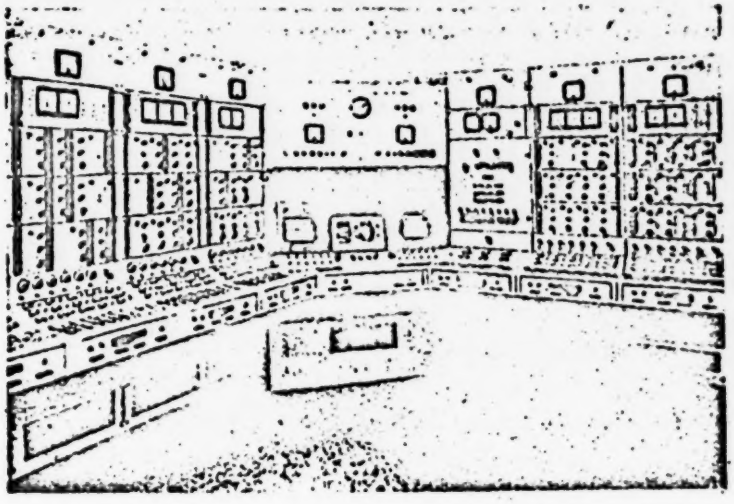
[5] Nuclear Power 1, 7, 303 (1956).

[6] Nucleonics 14, 7, R-10 (1956).

ELECTRONIC INSTALLATION SIMULATING THE PROCESSES IN ATOMIC ELECTRIC POWER STATIONS*

In Erith (County Kent, England), at the atomic energy research laboratory of the firm "Simon-Carves" (a division of the "General Electric Company"), there has been installed the largest and most highly developed electric model system of all those built in that country, to imitate precisely the processes in atomic electric power stations. This electronic machine, designed exclusively for research in the field of nuclear power, is one of the most important installations used for this purpose by the atomic energy administration of England, which is getting together in one center equipment for research, development, and testing, for the plan to construct numerous atomic electric power stations.

This modeling system will be used at first for the study of nonstationary processes occurring in atomic power stations, for example, as results of accidental emergencies, and will also be used for the study of the operation of automatic control systems for the stations.



General view of the electronic modeling apparatus.

* Electronic Engr. 28, 344, 437 (1956).

The use of electronic analogue devices makes it possible to predict quickly and accurately the behavior of individual components or groups of components in an atomic station without resorting to long and laborious calculations. As a simple example we can consider the change of power of the reactor brought about by a displacement of the control rods. Determination by a series of calculations of the curve characterizing this change requires several days. By regulating the voltage on one of the sections of the modeling apparatus the electric circuits of the section can be affected in a way corresponding to the effect on the reactor, and the result in the form of curves (drawn by an automatic tracer) is obtained in a matter of several minutes.

More complicated questions, relating to the interaction of several components of a power station, can be solved just as successfully, and the saving of time and labor can be even larger.

The flexibility and potentialties of the modeling apparatus have been the object of careful study during its development, so that the electronic machine makes it possible to carry out researches relating to more highly perfected types of reactors (in comparison with existing reactors with gaseous heat-removal materials and graphite modulators), which at the present time are in the planning stage.

An outstanding characteristic of the apparatus is the fact that it can be operated either as a single large machine or as two entirely independent smaller machines. The latter use permits the simultaneous study of two entirely different problems (if each does not require the use of the whole machine), which provides a considerable economy in time.

The modeling apparatus is installed in a special room in which the air pressure is kept somewhat above that of the atmosphere by means of ventilators placed above a suspended ceiling. This precaution makes it possible to avoid the intrusion of dust from neighboring rooms into the electrical system of the machine. Alongside the apparatus room there is laid out a workshop intended for the maintenance of the machine, the carrying out of minor repairs, and the testing of various components of the machine. In case of need this workshop can be used for the assembling and testing of various auxiliary circuits.

Many time-dependent processes occurring in an atomic power station last from fractions of a second up to several hours. Therefore the modeling apparatus is equipped with two continuously operating high-speed curve-plotting units, which provide curves from which the nature of the course of these processes in time can be judged. In addition, each section of the modeling apparatus is provided with an oscillograph, which can be used for selective study of the processes being simulated.

It must be noted that the simulation of time-dependent processes on this machine can be carried out at various speeds, i.e., with different time scales. For example, the study of the power regulating system and the emergency stopping of a reactor is of most value when run at actual speed, but the study of slowly varying processes, for example, the poisoning of a reactor by xenon, can best be done with shortening of the time.

The whole modeling apparatus consists of eight sections placed in the shape of the letter L. The control section is in the center (see picture). One of the sections simulates only processes going on in the reactor itself. The other six sections provide for the simulation of the operations of heat exchangers, gas blowers, the reactor regulating system, turbogenerators, and other components of an atomic power station.

The apparatus contains 100 amplifiers corrected for drift and 20 blocks of servoamplifiers. There are 84 amplifiers such that each can be used alternatively as an integrating amplifier, an integrator, or a servoamplifier. For the imitation of time lags two blocks are provided with a time delay of the output signal relative to the input signal.

The installation was carried out by the nuclear section of the firm "Elliott Brothers"; the design was prepared jointly with the firm "General Electric Company."

Yu. K.

EXPONENTIAL EXPERIMENTS WITH NATURAL URANIUM AND HEAVY WATER*

Exponential experiments on the system uranium-heavy water have been carried out in Sweden. A general view of the apparatus is shown in Fig. 1. Rods of natural uranium of diameters 2.00, 2.53, and 3.05 cm were immersed in heavy water. The interval of the hexagonal lattice was varied from 8 to 18 cm. A Ra-α-Be neutron source of activity 1.6 curies was placed in the lower part of the tank. Two long counters filled with BF_3 were used to measure the decrease of the neutron flux along the axis of the tank according to the law $\Phi = \Phi_0 e^{-\gamma z}$.

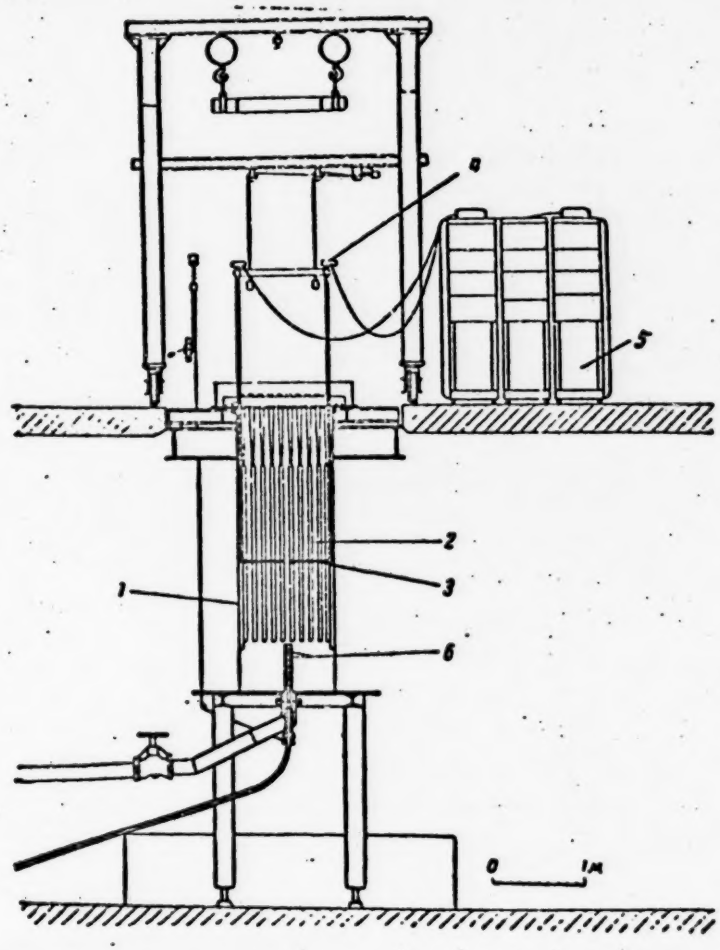


Fig. 1. Experimental arrangement.
1) Tank of heavy water; 2) uranium rod; 3) counter containing BF_3; 4) cathode repeater; 5) electronics unit; 6) Ra–Be source.

The Laplacian $B^2 = \left(\frac{2.405}{R}\right)^2 - \gamma^2$ is shown in Fig. 2 as a function of the volume ratio $\frac{V_m}{V_u}$ (R is the extrapolated radius of the tank; V_m is the volume of moderator; and V_u is the volume of uranium in a cell). In addition values have been calculated for the multiplication coefficient k, the thermal utilization coefficient f, and the square of the diffusion length for thermal neutrons, L^2. The values obtained are shown in the table.

* K. Persson, E. Blomsjö, M. Bustraan, and K. Meier, J. Nuclear Energy 3, 3, 188 (1956).

Rod diameter cm	Lattice interval cm	V_m/V_u	$(2.405/R)^2$ (m^{-2})	γ^2 (m^{-2})	B^2 (m^{-2})	L^2 (cm^2)	k	f
2.00	8	16.36	20.79	13.79	7.00	59.2	1.1256	0.9841
	9	21.05	20.84	12.90	7.94	77.1	1.1581	0.9818
	10	26.29	20.88	12.34	8.54	96.8	1.1890	0.9793
	11	32.07	20.92	12.52	8.40	118.0	1.2053	0.9768
	12	38.42	20.95	12.85	8.10	142.0	1.2190	0.9737
	14	52.75	21.01	13.98	7.03	196.9	1.2304	0.9668
2.53	8	9.75	20.72	17.66	3.06	39.0	1.0472	0.9862
	9	12.67	20.76	14.88	5.88	51.4	1.0992	0.9846
	10	15.95	20.81	13.26	7.55	65.2	1.1395	0.9829
	11	19.57	20.85	12.55	8.30	80.9	1.1680	0.9809
	12	23.53	20.88	12.22	8.66	97.8	1.1918	0.9788
	14	32.49	20.94	12.67	8.27	136.7	1.2181	0.9738
	16	42.82	20.99	13.46	7.53	182.8	1.2352	0.9681
3.05	10	10.60	20.75	15.57	5.18	47.5	1.0845	0.9846
	11	13.09	20.79	13.96	6.83	59.8	1.1211	0.9831
	12	15.81	20.83	12.75	8.08	73.0	1.1566	0.9814
	14	21.98	20.89	12.37	8.52	103.3	1.1928	0.9776
	16	29.09	20.95	12.72	8.23	139.0	1.2180	0.9731
	18	37.15	20.99	13.42	7.57	179.7	1.2331	0.9680

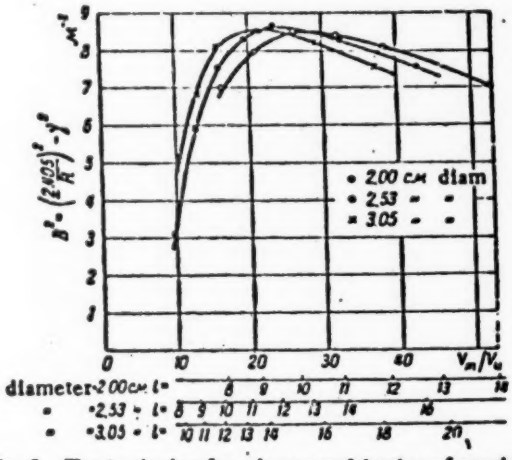


Fig. 2. The Laplacian for a hexagonal lattice of uranium rods of various diameters immersed in heavy water, as function of the ratio V_m/V_u.

From these data the effective number of secondary neutrons in the fission of natural uranium was found to be $\nu_{eff} = 1.311$, and the value of the effective resonance integral was found to be $(\int \sigma_a dE/E)_{eff} = 7.3 + 32\,S/M$ (S is the area of the uranium and M its mass).

P. K.

ON THE PROPERTIES OF URANIUM

There is very little information in the literature about the properties of uranium, so that some recently published data* are of interest, although they are mostly of a preliminary character.

1. The elastic modulus of the α-phase, determined from stress-deformation curves taken in compression, is $9.14 - 11.24 \cdot 10^3$ kg/mm^2. More recent determinations (Schoenfield, 1955) from stretching experiments gave $9.84 - 10.9 \cdot 10^3$ kg/mm^2.

2. Compressibilities of the α- and β-phases. The compressibility of the α-phase, $2.0 \cdot 10^{-6}$/bar at atmospheric pressure, was calculated from the formula $\frac{\Delta v}{v_p \Delta p}$, where $\Delta p \sim 1000$ kg/cm^2. According to other, possibly too high, data from measurements on metal not of "the highest purity," of density 18.93 ± 0.5 g/cm^3 at 25°C, the compressibility of the α-phase at 150°C is equal to $(2.5 \pm 0.5) \cdot 10^{-6}$/bar and that of the β-phase at 200°C is equal to $(2.3 \pm 1.0) \cdot 10^{-6}$/bar.

3. Thermodynamic properties.

a) Temperature rise in the transition alpha-beta:

0.00073°C/bar at 1000 bars
0.00102°C/bar at 5000 bars } (Spindler and Schoenfield)
0.00105°C/bar at 7000 bars (Bridgman).
(linear dependence)

b) Latent heat of the transition alpha-beta:

calculated from the Clausius-Clapeyron equation: ΔH = 925 cal/gm atom;

determined calorimetrically: ΔH = 985 cal/gm atom.

c) Specific heats of the α- and β-phases (C_p from preliminary measurements; C_v calculated).

Alpha-phase at 25°C: $C_p = 8.0$ cal/gm atom;
$C_v = 7.3$ cal/gm atom.

Beta-phase at 160°C: $C_p = 9.3$ cal/gm atom;
$C_v = 8.7$ cal/gm atom.

d) The surface tension, determined from radiography of a plutonium meniscus, is equal to 100 dynes/cm. The value theoretically predicted for this quantity from other physical constants (private communication from Taylor, Harwell) is 490-580 dynes/cm.

G. Z.

BEHAVIOR OF PLUTONIUM UNDER CYCLIC HEAT TREATMENT

In taking dilatometric curves of the cooling of plutonium one notes distinctly only the transition epsilon-delta; i.e., the behavior of this transition is anomalous. It has been studied** in Los Alamos and Harwell by

* Progress in Nuclear Energy (Pergamon Press, London-New York, 1956), Ser. V, Vol. I, p. 382.

** Progress in Nuclear Energy (Pergamon Press, London-New York, 1956), Ser. V, Vol. I, p. 379.

repeated cyclic heatings throughout the entire region of the solid phases and by cyclic heating and cooling within the regions of stability of the individual phases. Certain important properties have been established:

1. The apparent densities of the δ-phase obtained by heating and by cooling are decidedly different.

2. Related to the difference of density there is also a difference of the apparent coefficients of expansion of the δ-phase in heating and cooling.

3. The lattice parameters of the δ-phase determined from x-ray patterns of powder subjected to cyclic heat treatment did not show different values after heating and cooling and were usually in agreement with the density of the δ-phase as determined with the dilatometer during cooling.

4. Cyclic heat treatment with increasing cycles from the temperature of the ε-phase to low temperatures gives heating curves that return repeatedly along the path followed during cooling, if one leaves out of account the hysteresis of the transition temperatures, until the cycle extends to the gamma-beta transition, after which the heating curve becomes the normal one.

5. Repeated cyclic heat treatment from room temperature leads to an increasing reduction of the density and to a distortion of the shape of the surface, which is due, as with uranium and other metals, to the phase transitions. In Los Alamos the observations showed lengthening and shortening of the specimens, depending on the size and direction of the cycles.

6. If heating is stopped in the lower part of the region of existence of a phase there is no change of dimensions while the temperature is maintained constant. Heating and cooling of the δ-phase under these conditions gives a coefficient of expansion extremely close to that found by x-ray studies.

The hypothesis is put forward that after being cast and passing through the phase transitions in the cooling to room temperature the specimens of plutonium contain microcavities and also high temperature phases fixed in the cooling, so that the density is decreased. Microcavities arise for the most part in the cooling during the transitions between the complicated lattices of the γ- and β-phases. The presence of microcavities and the reduction of their volume in treatment by pressure is confirmed experimentally in the measurement of the elastic properties of plutonium by dynamical methods. On repeated heating the microcavities increase during the transition gamma-delta, so that the expansion as shown by a dilatometer commonly turns out greater than would be expected on the basis of crystallographic measurements. Transitions from the δ-phase into the δ'- and further into the ε-phase involves changes between comparatively simple lattices. This can be favorable to the removal of microcavities, which will be assisted by "sintering" which is to be expected at these relatively high temperatures. Certain indications of this phenomenon can be found in the dilatometer curves, since on the first heating the slope in the region of the ε-phase is considerably less than the slope on cooling. The slope on cooling gives a coefficient of expansion similar to that determined from x-ray studies and is found to be the same in repeated cycles in the delta-epsilon region both during cooling and also during heating.

If the microcavities mostly disappear in the epsilon phase, then it is understandable that cooling from the ε-phase to the gamma-beta phase transition proceeds on a reversible path close to that corresponding to crystallographic data.

There is agreement between these ideas and the fact that the expansion of plutonium on heating differs decidedly from specimen to specimen and even in different experiments with one and the same specimen. Such behavior can be expected because of differences in the grain structure and the influence of impurities. The behavior during cooling is more uniform, although it is still subject to the observed residual deformations of the surface.

An actual compression of the δ-phase during heating can lead to the closing up of the microcavities that have opened in the gamma-delta transition and can thereby increase the negative coefficient of expansion.

G. Z.

LOWERING OF THE STANDARD OF OCCUPATIONAL EXPOSURE TO RADIOACTIVE RADIATION IN THE U. S. A.*

After prolonged study a national committee on problems of protection from radiation of the National Bureau of Standards of the U.S.A. on December 8, 1956 adopted new allowable standards of occupational exposure to radioactive radiation.

The new standards limit the occupational exposure in industry to an average of 5 roentgens per person during a year and to approximately 280 roentgens during one's working life (up to the age of 70); this corresponds to one third of the allowable dose accepted at the present time. These new rules will be introduced into the procedures used by the Atomic Energy Commission and by atomic industry in the planning of factories and the setting of working conditions.

In his report at the winter meeting of the American Society of Atomic Energy, Taylor, the chairman of the committee, emphasized that so far as it is a question of effects that can be observed in the individual himself, the present allowable maxima appear to be safe ones.

But as a further measure of precaution against effects that may go unobserved – such as possible unnoticed decrease of the duration of life of the individual himself, and genetic damage capable of being transferred from generation to generation – it was decided to lower the standards by two thirds as applied to the whole lifetime of a person.

The main purpose of the lowering of the allowable dose is to decrease the irradiation of a person during his most productive years (from the genetic point of view). At the same time these new rules, which assure complete protection to the individual, must not hinder the development of atomic industry, and small departures from them should not influence the continuance of work by an individual in a given field.

Proceeding on the assumption that in the U.S.A. a person may enter work in an atomic industry at the age of eighteen years, the committee has developed formulas for determining the allowable cumulative level of radiation for a person of a given age:

$$5\,(n-18) \text{ roentgens,}$$

where n is the person's age.

Although this dose increases with the age, reaching 260 roentgens at 70 years of age, it is intended in the new rules that the maximum irradiation during a year must not exceed 15 roentgens and the average irradiation over the whole period of work must not exceed 5 roentgens per year.

The question of allowable doses arises only in relation to people who are occupationally concerned with radiation. It is assumed that in the U.S.A. such people do not number more than 500,000, i.e., 0.33% of the whole population. Nothing is said in the new rules concerning questions connected with radioactive fallout from hydrogen bombs or with maximum allowable levels of irradiation for the whole population.

In the general opinion of scientists the maximum exposure to radiation for the whole population must be considerably lower than the occupational maximum.

The allowable level of occupational exposure to radiation effective at present in the U.S.A. is 0.3 roentgens per week "during the course of the whole period of work." At such a level a person could receive during 40 years of work a dose of radioactive irradiation of 600 roentgens. According to the new standards this dose is lowered to 200 roentgens.

S. L.

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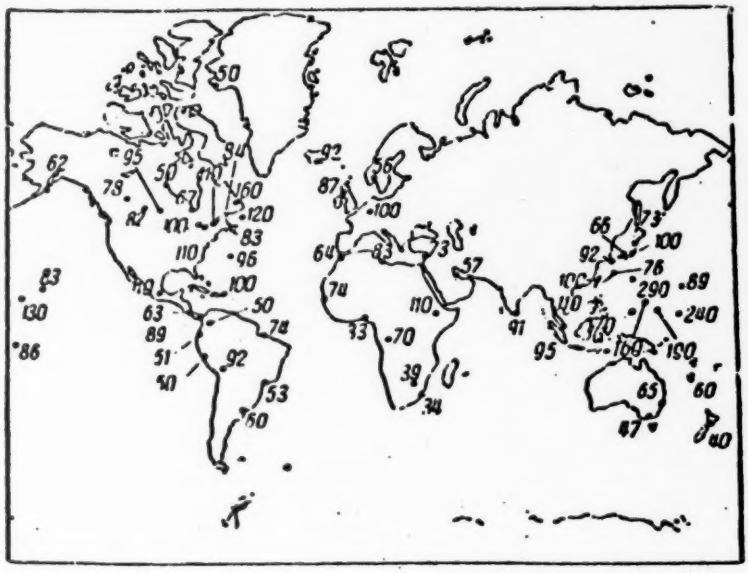
* Communication of the agency United Press from Washington, December 10, 1956.

DATA ON THE CONTAMINATION OF THE EARTH'S SURFACE BY RADIOACTIVE FALLOUT *

The Atomic Energy Commission of the U.S.A. has organized a broad network of stations for observation of radioactive fallout. Tests are carried on at 88 stations, including 26 stations in the territory of the U.S.A. The present communication summarizes the data obtained in the U.S.A. and abroad in September, 1955.

The full set of data from the control stations make it possible to give an estimate of the total surface contamination by a mixture of fission fragments and in particular by Sr^{90} (in millicuries per square mile) and of the total dose of γ-radiation (in millirads) from radioactive dust.

The data on the total contamination by fission fragments on January 1, 1956 are shown on the schematic chart of the world (see diagram).



Map of the distribution of contamination by mixed fission products in the vicinities of the control stations on January 1, 1956 (in millicuries per square mile).

As could be expected, the largest amounts of fallout of fission fragments were observed in the region of Nevada and in the test region in the Pacific Ocean. The greatest amount of deposition out of all those registered (Grand Junction, State of Colorado) amounted on January 1, 1956 to 740 millicuries per square mile. At all other places the values were lower, with the most intensive fallout (290 millicuries per square mile) registered at Iwo Jima in the western part of the Pacific Ocean. The smallest values – 33 millicuries per square mile – are found in Lagos, Nigeria (Africa). The distribution of mixed fission products is characterized, with some slight exceptions, by a high degree of uniformity in a given region, and the findings at neighboring stations do not differ by more than ± 50 %.

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* Science 1956, 3215, 251.

Contamination with Sr^{90} outside the borders of the U.S.A. ranges from 1.8 millicuries (Peru) to 24 millicuries per square mile (Iwo Jima). In the U.S.A. they range from 2.1 (San Francisco) to 23 millicuries (Salt Lake City). For most regions where the network of stations exist one can take for the average amount of deposition of Sr^{90} 2-5 millicuries per square mile.

The average values of the γ-ray dose in the U.S.A. are higher than in other parts of the world. The range of these values for the U.S.A. is relatively narrow, from 6 to 40 millirads, with the exception of Salt Lake City (160 millirads) Grand Junction (120 millirads) and Albuquerque (110 millirads). The typical dose for the Eastern States of the U.S.A. is from 15 to 20 millirads with somewhat higher values in the Middle West and somewhat lower ones on the West coast.

The total γ-ray dose at points outside the boundaries of the U.S.A. ranges from 4 to 23 millirads with the exception of some Pacific islands where it reaches some 150 millirads.

Since Sr^{90} is produced by the decay of Kr^{90}, an inert gas having a half-value period of 33 seconds, part of it is formed at a relatively late stage of the existence of the fireball. The relative content of Sr^{90} in the dust particles varies, in particles settling not far from the place of the explosion, Sr^{90} is present to only half the proportion that it occurs in particles settling at great distances.

The estimate of the size of the γ-ray dose, like the estimate of the Sr^{90} was made by calculation on the basis of data on the fallout of mixed fission products. It must be noted that the assumption on which the calculation of the γ-ray dose is based can lead to somewhat overestimated values. The stated quantities do not take into account the effect of screening and the influence of weather conditions (blowing away, washing out, etc.) which are not subject to quantitative treatment. For city populations the actual dose must be considerably reduced on account of the fact that substances falling to the surface were soon washed off into the gutters and the sewage system. For this reason it is very likely that the actual dose for a city population does not exceed 10% of the values obtained here.

The rural population is less protected by buildings and does not have the advantage of the removal of contamination by the sewage water. Nevertheless the actual condition of things never corresponds to an infinite level surface, which has been assumed in the calculations, and some reduction of the dose occurs here too because of the unevenness of the surface, the plowing of fields, and also by other factors.

In estimating the average dose of γ-radiation per individual of the world's population one can take the value of 10 millirads for the last four years. This quantity is very small in comparison with the dose of γ-radiation received by the population of the whole world from natural sources of radiation which comes to a value of the order of 75 millirads per year or 300 millirads for the 4 years. On this basis the dose of γ-radiation from radioactive fallout amounts to 3% of the average γ-ray dose from natural sources. Thus, even the maximum theoretical dose from fallout in the given time amounts to only a negligible fraction added to the dose of γ-radiation received because of natural radioactivity, and this slight addition is much smaller in amount than the fluctuation of the natural background.

The deposition of Sr^{90} in significant quantities can be compared with local natural deposits of radium, with which strontium has much in common in its chemistry. In the upper layer of soil of thickness 30 cm radium is present in quantities of approximately 1000 millicuries per square mile. This radium, like other elements, is assimilated by living organisms. The organism of an adult inhabitant of the U.S.A. contains approximately 10^{-4} microcuries of radium. This quantity amounts to 1/1000 of the maximum admissible content of radium, which is equal to 0.1 microcurie and less than 1/10,000 of the quantity which can produce injury to a human being.

Naturally occurring radium and the Sr^{90} contained in radioactive fallout are somewhat different in their properties, and this can influence their capability of passing from the soil into biological systems. One must however note the insignificant proportion of Sr^{90} in the soil in comparison with radium, and in particular in comparison with the very large concentrations of radium in some places, which nevertheless appear not to be dangerous.

A more direct method of estimating the values of Sr^{90} fallout consists in measuring the amount of this isotope not only in the soil but also in vegetation, animal organisms and food products. The concentration of this isotope in food depends on the content of calcium, so that it can best be expressed by the activity of the

Sr^{90} per 1 g calcium. From these calculations it has been found that milk in the U.S.A. at the beginning of 1956 contained about 3 micromicrocuries of Sr^{90} per gram of calcium. The usually accepted value for the maximum permissible content in the skeleton of an adult human being is 1 microcurie of Sr^{90}. The skeleton contains approximately 1000 g of calcium; thus the maximum permissible concentration must be 1000 micromicrocuries per gram of calcium, which exceeds by a factor of 350 the present observed concentration in milk.

According to data from the National Academy of Sciences, studies made on children show a measureable content of radioactive strontium in the organism. But the amount of it is entirely negligible – about 1/1000 of that which is accepted as the maximum permissible dose.

S. L.

BRIEF REPORTS

U. S. A. The firm "Martin Company" is building in Baltimore (State of Maryland) a portable reactor of power 1000 kw. for the Dominican Republic. This reactor, weighing 9.6 tons, can be packed for transportation to the place of assembly into eight aluminum containers (the largest of these is of dimensions 2.7 × 2.7 × 9 m). The assembly of the reactor requires not more than 3 days. As fuel the reactor uses 5000 ring-shaped fuel elements of slightly enriched uranium; the heat-transfer medium is water. (Engineer 202, 5261, 753, 1956).

U. S. A. The assembly of the Army's portable power reactor (Atomic Energy No. 1, 113,* 1956) is being completed at the construction site in Fort Belvoir (State of Virginia). The radiation shield has already been assembled and also the turbo-generator and the condenser; the radiological testing for defects in the welded seams of the protective case of the reactor has been completed and equipment has been set up for building the secondary heat-removal circuit and for welding the seams. The reactor tank and the pressure regulator for the circuit are being built by the firm "Alco" in Dunkirk (State of New York) and the active zone of the reactor is being constructed in Oak Ridge. It is expected that the reactor will be brought to the critical condition by March, 1957. After the reactor has been tested at various powers the operation at full power is intended to be begun in July, 1957. (Nucleonics 14, 8, R-11, 1956; Nucleonics 14, 9, R-9, 1956).

U. S. A. The Westinghouse Company has developed a method of visual observation and study of the pattern of mechanical stresses arising from high pressure in thick-walled steel jackets to contain reactors. Because of the presence of a large number of inserts, flanges, tubes, etc., the pattern of distribution of mechanical stresses in such high-pressure vessels is complicated, and to make a complete calculation of these stresses for such containers is very difficult. The proposed method makes it possible to observe these stresses directly. It is based on the use of an optical method of investigating stresses in models made of a special plastic. (J. of Colliers 5, 11, 1956).

U. S. A. In one of the laboratories of the "National Carbon" company (Parma, State of Ohio) there has been developed a method for "welding" graphite. This method is based on the use of high pressure to prevent the vaporization of graphite when heated to high temperatures. The "welding" of the graphite makes it possible to prefabricate the parts of a graphite moderator for a nuclear reactor, which at present are assembled from separate and disconnected graphite blocks. (Science News Letter 70, 13, 199, 1956).

U. S. A. In the course of experiments conducted in one of the laboratories of the Navy Department a temperature exceeding a million degrees Celsius has been attained. This research is regarded as a contribution to the problem of using reactions of synthesis for peaceful purposes, since for the production of a controlled synthesis reaction temperatures of this order are required (communication of the agency "France Presse" December 4, 1956).

England. The company "General Electric" has announced its intention of constructing for the power administration of southern Scotland "the most powerful atomic electric station in the world."

* Original Russian Pagination. See C. B. translation pagination.

The station will have two reactors of the Calder Hall type. The generators at the station will have a total power of 360,000 kw, of which 40 to 60 thousand kw will be used for the requirements of the station itself. The saving of coal resulting from the introduction of this station will amount to one million tons per year. (Communication of the agency TASS from London).

Finland. Arkki Aalto, the president of a Finnish company for atomic power, has announced that by 1960 there will be constructed in Finland an atomic reactor, and that negotiations have already begun with England and the U. S. A. The company is mainly interested in obtaining steam for the paper industry. A similar interest is also being expressed in such countries as Norway, Sweden, and Canada, where the timber working industry plays a prominent part in the economy of the country. (Nuclear Engineering 1, 7, 310, 1956).

France. The French transatlantic company "French Line" has announced its plans to construct by 1960 a transatlantic passenger steamer "France" of displacement 55,000 tons (speed 30 knots). The engine room of this vessel will be planned to include the replacement of 8 steam boilers by nuclear reactors, in case economical ways are found for using atomic energy for ocean transport. (Nucleonics 14, 9, R-11, 1956).

Japan. The ministry of transport of Japan is planning to construct by 1966 two atomically driven vessels of power 20 to 30 thousand horse power. One of the power plants will be constructed in Japan and the second will be obtained from abroad. (Nucleonics 14, 9, R-11, 1956).

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